Exercise 1

Use ¬, →, ∧ and ∨ to express the following declarative sentences in propositional logic; in each case state what your respective propositional atoms p, q, etc. mean:

- If the sun shines today, then it won’t shine tomorrow.
- Robert was jealous of Yvonne, or he was not in a good mood.
- If the barometer falls, then either it will rain or it will snow.
- If a request occurs, then either it will eventually be acknowledged, or the requesting process won’t ever be able to make progress.
- Cancer will not be cured unless its cause is determined and a new drug for cancer is found.
- If interest rates go up, share prices go down.
- If Smith has installed central heating, then he has sold his car or he has not paid his mortgage.
- Today it will rain or shine, but not both.
- If Dick met Jane yesterday, they had a cup of coffee together, or they took a walk in the park.
- No shoes, no shirt, no service.

Exercise 2

The formulas of propositional logic below implicitly assume the binding priorities of the logical connectives.

Reinsert as many brackets as possible to show your understanding of the binding properties:

For example, given p ∧ q → r, change it to (p ∧ q) → r since ∧ binds more tightly than →.

- ¬p ∧ q → r
- (p → q) ∧ ¬(r ∨ p → q)
- (p → q) → (r → s ∨ t)
- p ∨ (¬q → p ∧ r)
- p ∨ q → ¬p ∧ r
- p ∨ p→ ¬q
- p ∨ q ∧ r

• ¬ binds more tightly than ∨ and ∧,
• ∨ and ∧, bind more tightly than →.
• Implication → is right-associative: expressions of the form p → q → r denote p → (q → r).
Exercise 3

□ Develop truth tables for the following:
   □ p ^ q
   □ p v q
   □ ¬ p
   □ p → q

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>A → B</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Exercise 4

□ Write the following as sequent
   □ If the train arrives late and there are no taxis at the station, then John is late for his meeting. John is not late for his meeting. The train did arrive late.

□ If it is raining and Jane does not have her umbrella with her, then she will get wet. Jane is not wet. It is raining.

p ∧¬q → r, ¬r, p |¬ q
Exercise 5

- Prove by deduction:
  - \((p \land q) \land r, s \land t \models q \land s\)

Exercise 6

- Prove by deduction:
  - \(p \rightarrow (q \rightarrow r), p, \neg r \models \neg q\)