Data Structures and Algorithms – CS2402

Exercises on Algorithms Analysis (2)

Exercise 1 *Show that* $\log_c(ab) = \log_c(a) + \log_c(b)$.

Solution: Let $k = \log_c(ab)$*, then by definition of the logarithm,* $ab = c^k$.
But now, you can also express a *and* b *in terms of a power of* c*, separately. Indeed, if* $k_1 = log_c(a)$ *and* $k_2 = log_c(b)$*, then* $a = c^{k_1}$ *and* $b = c^{k_2}$.
As a result: $a.b = c^{k_1}.c^{k_2}$*. Since we also know that* $ab = c^k$*, it comes that* $c^{k_1}.c^{k_2} = c^k$*, which is* $c^k = c^{k_1+k_2}$.
We deduce from this that $k = k_1 + k_2$*, and therefore* $\log_c(ab) = \log_c(a) + \log_c(b)$. □

Exercise 2 *Show that* $\log_b(a^n) = n \log_b(a)$.

Solution: Let $k = \log_b(a)$*. Then, by definition of the logarithm,* $a = b^k$. $a^n = (b^k)^n = b^{nk}$*. Therefore,* $\log_b a^n = nk = n. \log_b(a)$. □

Exercise 3 *Show that* $\log_b(a) = \frac{\log_c(a)}{\log_c(b)}$.

Solution: Let $a = b^k$, $a = c^l$ *and* $b = c^m$*. Then,* $a = (c^m)^k = c^l$.
As a result, $l = m.k$*, and therefore:* $\log_c(a) = \log_b(a) \log_c(b)$. □

Exercise 4 *Show that* $\log_b(a) = \frac{1}{\log_a(b)}$.

Solution: Let $k = \log_b(a)$*, then* $a = b^k$ *which means that* $b = a^{\frac{1}{k}}$*. As a result,* $\log_a(b) = \frac{1}{k}$*, and* $\log_b(a) = \frac{1}{\log_a(b)}$. □

Exercise 5 *Show that* $a^{\log_b(n)} = n^{\log_b(a)}$.

Solution: Let $k = \log_b(n)$ *and* $l = \log_b(a)$*. Therefore,* $n = b^k$ *and* $a = b^l$.
$b^{l.k} = (b^l)^k = (b^k)^l$ *which means that* $n^l = a^k$. □

Exercise 6

```
sum=0;
for (int i = 0; i < n^2; i* = 5)
    sum++;
```

The running time function of this fragment of code is as follows:

sum=0;	*1 step*
int i=0;	*1 step*
per valid loop:	
$i < n^2$	*1 step*
$i* = 5$	*1 step*
$sum++:$	*1 step*
	= *3 steps per valid loop*
	× *number of valid loops* $= \log_5(n^2)$
	$= 3 \times \log_5(n^2)$
exit condition: $i < n^2$ *when* $i \geq n^2$	*1 step*
Total steps	$= 3 + 3 \times \log_5(n^2)$

Therefore the big-Oh of this function is: $\mathcal{O}(\log(n))$, *because* $3 + 3 \times \log_5(n^2) = 3 + 6 \times \log_5(n)$.

Exercise 7

$$n^2 \log(n) \succ n^2 \succ n \log(n) \succ n \succ \log(n)$$