

Logical Foundations of CS – CS5303

Exercises on Predicate Logic

1 Make deductions

Exercise 1 Consider the following statements:

1. The only animals in this house are cats.
2. Every animal who likes to stare at the moon is a domestic animal.
3. When I hate an animal, I avoid it carefully.
4. No animal is carnivore, unless it hangs out at night.
5. No cat fails to kill mice.
6. No animal likes me, except for those in this house.
7. Kangaroos are not able to become domestic animals.
8. No non-carnivore animal can kill a mouse.
9. I hate animals who don't like me.
10. Animals who hang out at night like to stare at the moon.

Following from these statements, can we deduce that:

I carefully avoid kangaroos.

In order to answer this question,

1. use your own reasoning;
2. use the Venn diagrams as done in class on Tuesday.

2 Equivalent formulas

Exercise 2 Please connect the following formulas by pair of equivalent formulas. Informally justify your choices.

1. $(\forall v \phi) \wedge (\forall v \psi)$

2. $\exists v \phi$
3. $\neg \exists v \neg \phi$
4. $\exists v (\phi \vee \psi)$
5. $\forall v (\phi \wedge \psi)$
6. $\forall v \phi$
7. $\exists v (\phi \vee \psi)$
8. $\neg \forall v \neg \phi$

Exercise 3 Suppose that v is not a free variable in ψ . Same thing as in previous exercise.

1. $(\exists v \phi) \wedge \psi$
2. $\exists v (\psi \rightarrow \phi)$
3. $\forall v (\phi \wedge \psi)$
4. $\psi \rightarrow (\exists v \phi)$
5. $\exists v (\phi \wedge \psi)$
6. $(\forall v \phi) \wedge \psi$

Exercise 4 Here are two rules (from the top, you can deduce the bottom, and reversely):

$$1). \frac{\phi}{\forall v \phi} \qquad 2). \frac{\phi(r)}{\exists v \phi}$$

Give an informal explanation of these two rules.