Logical Foundations of CS – CS5303
$\mathfrak{E}$xercises on $\mathfrak{I}$nduction

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1 Proof by induction

Exercise 1 *Let BT be the set of binary trees. Let h, n and l denote respectively the height of the tree, its number of nodes, and its number of leaves. Using the inductive definition of AB, show that:*

- $\forall x \in AB,\ n(x) \leq 2^{h(x)} - 1$
- $\forall x \in AB,\ f(x) \leq 2^{h(x)-1}$

2 Inductively defined functions

Exercise 2 *The mirror or reverse of a word $u = a_1.a_2.\ldots.a_n$ is the word $u^R = a_n.\ldots.a_2.a_1$. Give an inductive definition of the mirror/reverse function:*

$$u = a_1.a_2.\ldots.a_n \mapsto u^R = a_n.\ldots.a_2.a_1.$$

Exercise 3 *Give an inductive definition of n the number of nodes of a binary tree, and of l the number of leaves of a binary tree.*

Exercise 4 *Give the inductive definition of the pre-order traversal of a binary tree.*

3 Set closure

Exercise 5 *Let $E = \mathbb{N}$, and $C(A) = \{n+m \mid n \in A,\ m \in A\}$. Let $k\mathbb{N} = \{kn \mid n \in \mathbb{N}\}$. Show that: if $A \subseteq k\mathbb{N}$ then $\widehat{C}(A) \subseteq k\mathbb{N}$.*