Logical Foundations of CS – CS5303

Quiz 1 — 50 points / 50 minutes

Important note: Please make sure that you justify your answers, and that your answers are readable. All non-justified answers will be graded half the points, all unreadable answers will be graded 0. Besides 3 points will be given for the clarity of your answers, and their presentation.

1 Sets (Total: 9 points)

Exercise 1 (5 points) *Let A, B, C be three subsets of E. Show that:*

$$A \cap \overline{B} = A \cap \overline{C} \Leftrightarrow A \cap B = A \cap C$$

Exercise 2 (4 points) *Let A be a subset of E, and let $(B_i)_{i \in I}$ a family of subsets of E. Show that:*

$$A \cup (\bigcap_{i \in I} B_i) = \bigcap_{i \in I} (A \cup B_i)$$

2 Functions (Total: 12 points)

Exercise 3 (4 points) *Let $f : A \to B$ be an application. Show that:*

$$f \text{ injective} \Rightarrow \forall X \subseteq A,\ f^{-1}(f(X)) = X$$

Exercise 4 (8 points) *Let us consider two applications $f : A \to B$ and $g : B \to C$. Show that:*

1. $g \circ f$ *injective* $\Rightarrow$ f *injective*
2. $g \circ f$ *surjective* $\Rightarrow$ g *surjective*

3 Cardinality (Total: 6 points)

Exercise 5 (6 points) *Show that the set $\mathbb{N} \times \mathbb{N}$ is countable.*

4 Relations (Total: 20 points)

Exercise 6 (20 points) *Let E be a finite set: $E = \{e_1, \ldots, e_n\}$, et let $\mathcal{R}$ be a binary relation over E. We can represent $\mathcal{R}$ using a $n \times n$ matrix, say $\mathcal{M}_{\mathcal{R}}$, whose elements*

belong to $\{0,1\}$*, and are defined as follows:*

$$m_{i,j} = \begin{cases} 1 & \textit{if } e_i \ \mathcal{R} \ e_j \\ 0 & \textit{otherwise} \end{cases}$$

1. *Describe* $\mathcal{M}_{\mathcal{R}}$ *in case:*
   (a) $\mathcal{R}$ *is a symmetrical relation*
   (b) $\mathcal{R}$ *is a reflexive relation*
2. *Suppose you have two relations over* E*, say* $\mathcal{R}$ *and* $\mathcal{R}'$*, how can you compute:* $\mathcal{M}_{\mathcal{R}^{-1}}$, $\mathcal{M}_{\overline{\mathcal{R}}}$.