Logical Foundations of CS – CS5303
Quiz 4 Solution

Exercise 1 (Skolem and CNF forms) (1 + 2 + 4 + 4 points) *Put the following formulas into Skolem form, and then CNF form.*

1. $\forall x,\ \exists y,\ father(y, x)$
   Skolem form: $father(f(x), x)$
   CNF=Skolem form

2. $\exists x,\ (person(x) \wedge (\forall y,\ (person(y) \rightarrow \exists z,\ (person(z) \wedge \neg equal(y, z) \wedge ((knows(y, x)) \vee (knows(z, x)))))))$
   Prenex form: $\exists x \forall y \exists z,\ (person(x) \wedge (person(y) \rightarrow (person(z) \wedge \neg equal(y, z) \wedge (knows(y, x) \vee knows(z, x)))))$
   Skolem form: $person(x_0) \wedge (person(y) \rightarrow (person(f(y)) \wedge \neg equal(y, f(y)) \wedge (knows(y, x_0) \vee knows(f(y), x_0))))$
   CNF: $person(x_0)\ \wedge\ (\neg person(y) \vee person(f(y)))$
   $\wedge\ (\neg person(y) \vee \neg equal(y, f(y)) \vee person(f(y)))$
   $\wedge\ (\neg person(y) \vee knows(y, x_0) \vee person(f(y)))$
   $\wedge\ (\neg person(y) \vee \neg equal(y, f(y)))$
   $\wedge\ (\neg person(y) \vee knows(y, x_0) \vee \neg equal(y, f(y)))$
   $\wedge\ (\neg person(y) \vee person(f(y)) \vee knows(f(y), x_0))$
   $\wedge\ (\neg person(y) \vee \neg equal(y, f(y)) \vee knows(f(y), x_0))$
   $\wedge\ (\neg person(y) \vee knows(y, x_0) \vee knows(f(y), x_0))$

Exercise 2 (Translations of English sentences) (1 + 1 + 2 + 4 + 4 + 1 + 1 points) *Translate the following sentences, and answer the questions that are asked below (if any).*

1. *There is a penguin that can fly.*
   $\exists x, (penguin(x) \wedge canfly(x))$

2. *If anyone wakes up late and this is a workday, then this person is late at work.*
   $\forall y \forall z, ((person(y) \wedge day(z)) \rightarrow ((wakes\_up\_late(y) \wedge work\_day(z)) \rightarrow late\_at\_work(y)))$

3. *There are good basket-ball players that are not tall.*
   $\exists x, (person(x) \wedge basketball\_player(x) \wedge \neg tall(x))$
   $\not\forall x, ((person(x) \wedge basketball\_player(x)) \rightarrow tall(x))$

4. *It is necessary to be motivated to succeed at College.*
   $\forall x, (person(x) \rightarrow (succeed\_at\_College(x) \rightarrow motivated(x)))$ *(1)*

   - *Can you deduce from this statement that if you are not motivated, you will fail at College? Prove your answer.*
     Here what we want to prove is that: $\forall x, (person(x) \rightarrow (\neg motivated(x) \rightarrow \neg succeed\_at\_College(x)))$ *(2).*
     Formula (1) is equivalent to:

$person(x) \rightarrow (succeed\_at\_College(x) \rightarrow motivated(x))$
$\equiv \neg person(x) \vee \neg succeed\_at\_College(x) \vee motivated(x)$
$\equiv \neg person(x) \vee \neg succeed\_at\_College(x) \vee \neg\neg motivated(x)$
$\equiv \neg person(x) \vee \neg(\neg motivated(x)) \vee \neg succeed\_at\_College(x)$
$\equiv \neg person(x) \vee ((\neg motivated(x)) \rightarrow \neg succeed\_at\_College(x))$
$\equiv person(x) \rightarrow ((\neg motivated(x)) \rightarrow \neg succeed\_at\_College(x))$
$\equiv \forall x, (person(x) \rightarrow ((\neg motivated(x)) \rightarrow \neg succeed\_at\_College(x))) \equiv (2)$

5. *It is sufficient to be tall to be a good basket-ball player.*
   $\forall x, ((person(x) \wedge tall(x)) \rightarrow good\_basketball\_player(x))$ *(1)*

   - *Can you deduce from this statement, that if you are a good basket-ball player, you are tall? Prove your answer. Here what we want to prove is that:* $\forall x, ((person(x) \wedge good\_basketball\_player(x)) \rightarrow tall(x))$ *(2)*
     $\equiv (person(x) \wedge good\_basketball\_player(x)) \rightarrow tall(x)$
     $\equiv \neg person(x) \vee \neg good\_basketball\_player(x) \vee tall(x)$
     On the other hand, formula (1) is equivalent to:
     $(person(x) \wedge tall(x)) \rightarrow good\_basketball\_player(x)$
     $\equiv \neg person(x) \vee \neg tall(x) \vee good\_basketball\_player(x)$
     $\not\equiv (2)$.
     So, from formula (1), we cannot make the above deduction.

6. *Not all penguins can fly.*
   $\neg\forall x, (penguin(x) \rightarrow canfly(x))$

7. *No horse can fly.* $\forall x, (horse(x) \rightarrow \neg canfly(x))$

Exercise 3 (Reasoning) (4 + 4 points) *Are the following reasoning correct? Prove your answers, by translating the reasonings in FOL, and then by using a proving method/a deduction system.*

1. *When you wake up late, you are late at work and, if you work at Untel Company, you get a bad evaluation. Tom did not get a bad evaluation.*
   Can we deduce from the above that: he either did not wake up late or that he doesn't work at Untel Company?

   1. $\forall x, (person(x) \rightarrow (wake\_up\_late(x) \rightarrow (late\_at\_work(x) \wedge (work(x, Untel) \rightarrow (bad\_eval(x))))))$
   2. $\neg bad\_eval(Tom)$
   3. $person(Tom)$ *(implicit knowledge)*

   (1) is equivalent to: $(\neg person(x) \vee \neg wake\_up\_late(x) \vee late\_at\_work(x)) \wedge (\neg person(x) \vee \neg wake\_up\_late(x) \vee \neg work(x, Untel) \vee bad\_eval(x))$

   From this, we want to deduce that:
   4. $\neg wake\_up\_late(Tom) \vee \neg work(Tom, Untel)$

   So we instanciate (unify) (1) with $x = Tom$*, and apply resolution on the following reasoning: (1), (2), (3), ¬ (4), which translates into the following set of clauses:*
   $\{\neg person(Tom) \vee \neg wake\_up\_late(Tom) \vee late\_at\_work(Tom), \neg person(Tom) \vee \neg wake\_up\_late(Tom) \vee \neg work(Tom, Untel) \vee bad\_eval(Tom), \neg bad\_eval(Tom), person(Tom), wake\_up\_late(Tom), work(Tom, Untel)\}$

From $\neg person(Tom) \lor \neg wake\_up\_late(Tom) \lor late\_at\_work(Tom)$ *and* $person(Tom)$, *we deduce:* $\neg wake\_up\_late(Tom) \lor late\_at\_work(Tom)$. *The same way, we obtain:* $\neg wake\_up\_late(Tom) \lor \neg work(Tom, Untel) \lor bad\_eval(Tom)$.

Then from $\neg wake\_up\_late(Tom) \lor late\_at\_work(Tom)$ *and* $wake\_up\_late(Tom)$, *we deduce that* $late\_at\_work(Tom)$. *Similarly, we deduce that* $\neg work(Tom, Untel) \lor bad\_eval(Tom)$.

From $\neg work(Tom, Untel) \lor bad\_eval(Tom)$ *and* $\neg bad\_eval(Tom)$, *we deduce:* $\neg work(Tom, Untel)$.

And finally, using $work(Tom, Untel)$, *we deduce a contradiction.*

This means that it was correct to deduce that Tom either did not wake up late or doesn't work at Untel Company.

2. *If you win at the lottery, then if the amount is significant, you can stop to work. Tom still did not stop to work.*
   Can you deduce from the above that he did not win the lottery?

   1. $\forall x, \forall y, ((person(x) \land money(y)) \to (win\_lottery(x, y) \to (significant(y) \to stop\_work(x))))$
   2. $\neg stop\_work(Tom)$
   3. $person(Tom)$ *(implicit knowledge)*

   From this, we want to deduce that:
   4. $\neg win\_lottery(Tom, y)$ *(*$\equiv \forall y, \neg win\_lottery(Tom, y)$*).*

   Let us try and prove that: $(1) \land (2) \land (3) \land \neg(4)$ *leads to a contradiction, using resolution. We unify the above formulas, and obtain the following substitution:* $x = Tom$. *The set of clauses we consider is the following:*
   $\{\neg person(Tom) \lor \neg money(y) \lor \neg win\_lottery(Tom, y) \lor \neg significant(y) \lor stop\_work(Tom), person(Tom), \neg stop\_work(Tom), win\_lottery(Tom, y)\}$

   From the first clause and $person(Tom)$, *we deduce that:* $\neg money(y) \lor \neg win\_lottery(Tom, y) \lor \neg significant(y) \lor stop\_work(Tom)$.

   From this new clause and $\neg stop\_work(Tom)$, *we deduce that:* $\neg money(y) \lor \neg win\_lottery(Tom, y) \lor \neg significant(y)$.

   From this and $win\_lottery(Tom, y)$, *we deduce that:* $\neg money(y) \lor \neg significant(y)$.

   And we cannot make more clause resolutions. Which means that we do not reach a contradiction and the deduction that Tom did not win the lottery was not correct.

   Indeed, maybe he did not win the lottery, or he did and the amount of money was not significant enough so that he can quit his job.