Logical Foundations of CS – CS5303
Exercises on Predicate Logic 2

Exercise 1 *Translate the following sentences into predicate logic formulas.*

1. *Not every book is interesting.*
2. *Some books are interesting.*
3. *Some books are interesting and some are easy to read.*
4. *Every interesting book is easy to read.*
5. *Whenever a book is well written or easy to understand, it is expensive.*
6. *Not all expensive books are easy to understand.*
7. *A well written book is not necessarily easy to understand.*
8. *For a book to be easy to understand it is necessary that it is well written.*
9. *For every expensive book, there is a book that is not expensive but well written.*
10. *Not every red bee is large.*
11. *Every red bee likes every large flower.*
12. *Not everyone likes everyone else.*
13. *Not every nice person is happy.*
14. *Every nice person likes every happy person.*
15. *There is someone whom no one likes.*
16. *For any even number there is an odd number which is greater than that.*

Then, put the formulas into clausal form (i.e., prenex and then skolem, and CNF).

Exercise 2 *Formalize the following reasonings in predicate logic. Then prove that these reasonings are correct.*

1. *John, a student in this class, is 16 years old. Everyone who is 16 years old can get a driver's liscence. Therefore, someone in this class can get a driver's liscence.*
2. *Someboday in this class enjoys hiking. Every person who enjoys hiking also likes biking. Therefore, there is a person in this class who likes biking.*

3. *Every student in this class owns a personal computer. Everyone who owns a personal computer can use the Internet. Therefore, John, a student in this class, can use the Internet.*

4. *Everyone in this class owns a personal computer. Someone in this class has never used the Internet. Therefore, someone who owns a personal computer has never used the Internet.*

Exercise 3 *Determine whether each of the following arguments is valid. If an argument is correct, what rule of inference is being used? If it is not, what fallacy occurs?*

1. *If n is a real number with* $n > 1$*, then* $n^2 > 1$*. Suppose that* $n^2 \leq 1$*. Then* $n \leq 1$.

2. *If n is a real number with* $n > 1$*, then* $n^2 > 1$*. Suppose that* $n^2 > 1$*. Then* $n > 1$.

Solutions and other:

Check: http://andromeda.cs.odu.edu:28080/InferenceCheck/index.html

Translations:

1. Not every book is interesting: $\neg\forall x(book(x) \rightarrow interesting(x))$
2. Some books are interesting: $\exists x, (book(x) \wedge interesting(x))$
3. Some books are interesting and some are easy to read:
   $(\exists x, (book(x) \wedge interesting(x))) \wedge (\exists x, (book(x) \wedge easy - to - read(x)))$
4. Every interesting book is easy to read:
   $\forall x, ((book(x) \wedge interesting(x)) \rightarrow easy - to - read(x))$
5. Whenever a book is well written or easy to understand, it is expensive:
   $\forall x, ((book(x) \wedge (well - written(x) \vee easy - to - understand(x))) \rightarrow expensive(x))$
6. Not all expensive books are easy to understand:
   $\neg\forall x\ ((book(x) \wedge expensive(x)) \rightarrow easy - to - understand(x))$
7. A well written book is not necessarily easy to understand:
   $\neg\forall x, ((book(x) \wedge well - written(x)) \rightarrow easy - to - understand(x))$
8. For a book to be easy to understand it is necessary that it is well written:
   $\forall x, (book(x) \rightarrow (easy - to - understand(x) \rightarrow well - written(x)))$
   $\forall x, ((book(x) \wedge easy - to - understand(x)) \rightarrow well - written(x))$
9. For every expensive book, there is a book that is not expensive but well written:
   $\forall x, ((book(x) \wedge expensive(x)) \rightarrow (\exists y, (book(y) \wedge \neg expensive(y) \wedge well - written(y)))$
10. Not every red bee is large: $\neg\forall x((red(x) \wedge bee(x)) \rightarrow large(x))$
11. Every red bee likes every large flower:
    $\forall x, ((red(x) \wedge bee(x)) \rightarrow (\forall y, ((flower(y) \wedge large(y)) \rightarrow likes(x, y))))$
12. Not everyone likes everyone else:
    $\neg\forall x, (person(x) \rightarrow (\forall y, (person(y) \rightarrow likes(x, y))))$
13. Not every nice person is happy:
    $\neg\forall x, ((person(x) \wedge nice(x)) \rightarrow happy(x))$
14. Every nice person likes every happy person:
    $\forall x, ((person(x) \wedge nice(x)) \rightarrow (\forall y, ((person(x) \wedge happy(y)) \rightarrow likes(x, y))))$
15. There is someone whom no one likes:
    $\exists x, (person(x) \wedge (\forall y, (person(y) \rightarrow \neg likes(y, x))))$
16. For any even number there is an odd number which is greater than that. $\forall x, ((number(x) \wedge even(x)) \rightarrow (\exists y, (number(y) \wedge odd(y) \wedge greater(y, x))))$

Reasonings:

1. Let:
   - C(x) be: x is a student in this class,
   - S(x) be: x is 16 years old,
   - and D(x) be: x can get a driver's licence.

   The premises are C(John), S(John) and $\forall x, (S(x) \rightarrow D(x))$. Using universal instantiation and the last premise, $S(John) \rightarrow D(John)$ follows. Applying modus ponens to this conclusion and the second premise, $D(John)$ follows. Using conjunction and the first premise, $C(John) \wedge D(John)$ follows. Finally, using existential generalization, the desired conclusion, $\exists x, (C(x) \wedge D(x))$ follows.

2. Let:
   - C(x) be: x is in this class,
   - H(x) be: x enjoys hiking,
   - and B(x) be: x likes biking.

   The premises are $\exists x, (C(x) \wedge H(x))$ and $\forall x, (H(x) \rightarrow B(x))$. Using existential instantiation and the first premise $C(d) \wedge H(d)$ for some person d. From this by simplification H(d) is obtained. For that person d, by universal instantiation and the second premise, $H(d) \rightarrow B(d)$. Hence by modus ponens B(d) follows. Also by simplification from $C(d) \wedge H(d)$, C(d) is obtained. Using the conjunction on the last two conclusions, $C(d) \wedge B(d)$ is obtained. Then applying existential generalization to this $\exists x, (C(x) \wedge B(x))$ follows.

3. Let:
   - C(x) be: x is a student in this class,
   - P(x) be: x owns a personal computer,
   - and I(x) be: x can use the Internet.

   The premises are $\forall x, (C(x) \rightarrow P(x))$, $\forall x, (P(x) \rightarrow I(x))$, and C(John). Using universal instantiation and the first premise, $C(John) \rightarrow P(John)$ follows. Applying modus ponens to this conclusion and the last premise, P(John) follows. Using universal instantiation and the second premise, $P(John) \rightarrow I(John)$ follows. From the last two conclusions by modus ponens I(John) follows. Hence from this conclusion and the last premise by conjunction $C(John) \wedge I(John)$ follows.

4. Let:
   - C(x) be: x is a student in this class,
   - P(x) be: x owns a personal computer,
   - and I(x) be: x has used the Internet.

The premises are $\forall x, (C(x) \rightarrow P(x))$ and $\exists x, (C(x) \wedge \neg I(x))$. Using existential instantiation and the second premise $C(d) \wedge \neg I(d)$ for some person d. Hence by simplification C(d). For that person d, by universal instantiation and the first premise $C(d) \rightarrow P(d)$. From this and the previous conclusion, by modus ponens P(d). Also from $C(d) \wedge \neg I(d)$ by simplification I(d). Hence by conjunction $P(d) \wedge I(d)$. Hence by existential generalization $\exists x, (C(x) \wedge I(x))$.

Check Reasoning:

1. If n is a real number with $n > 1$, then $n^2 > 1$. Suppose that $n^2 \leq 1$. Then $n \leq 1$. Valid argument using modus tollens:

$$\neg(n^2 > 1) \rightarrow \neg(n > 1) \equiv (n^2 \leq 1) \rightarrow (n \leq 1)$$

2. If n is a real number with $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$. Fallacy of affirming the conclusion.