

Artificial Intelligence

Joint class 5314 / 4320

Probabilities

Main resources for this lecture:

- the Textbook of this class: chapter 13.
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Objectives

1. **Be aware that you can deal with belief when knowledge is not available:**
Therefore you may not have only true or false statements: but belief ($\in [0, 1]$) that a statement holds or not.
e.g., $P(\text{rain in El Paso})=0.3$
 $P(\text{cavity} \mid \text{toothache}) = 0.8$: in this case it does not mean any thing about the truth value (0 or 1) of the statement $\text{cavity} \rightarrow \text{toothache}$
Probability is what the agent believe about the statements (its degree of belief), not the result of an observation of the world.

2. **Know:** discrete variables, joint probability distribution, etc.

3. **Know the prior and conditional probabilities:**

$$P(a|b) = \frac{P(a \wedge b)}{P(b)}$$

where $P(a|b)$ is the probability of a given b .

4. **Know the axioms of probability:**

- (a) $P(a) \in [0, 1]$
- (b) $P(\text{true}) = 1$: means valid, unquestionable
 $P(\text{false}) = 0$: means unsatisfiable
- (c) $P(a \vee b) = P(a) + P(b) - P(a \wedge b)$

5. From the above, be able to deduce that: $P(\neg a) = 1 - P(a)$

$$\begin{aligned}P(a \vee \neg a) &= P(a) + P(\neg a) - P(a \wedge \neg a) \\&= P(a) + P(\neg a) - P(\text{false}) \\&= P(a) + P(\neg a) - 0 \\&= P(a) + P(\neg a) \\P(\text{true}) &= P(a \vee \neg a) \text{ using logic rule: } 1 \equiv a \vee \neg a \ (\forall a) \\&= P(a) + P(\neg a) \\&= 1\end{aligned}$$

Therefore:

$$\begin{aligned}P(a) + P(\neg a) &= 1 \\P(\neg a) &= 1 - P(a)\end{aligned}$$

Remarks

1. Consider a discrete variable $D = \langle d_1, \dots, d_n \rangle$:

$$\sum_{i=1}^n P(D = d_i) = 1$$

You must be able to prove (at least justify) this.

+ to prove that this is also true for joint probability distributions.

2. Prove that: $a \leftrightarrow \bigvee$ (all events such that a holds)

+ all atomic events being mutually exclusive, we have:

$$P(a) = \sum_{e_i \in e(a)} P(e_i)$$

Exercises

1. Work on Section 13.4, try and make deductions (cf. 13.6).

2. Do exercise 13.16