Interferogram Analysis using Active Instance-Based Learning

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Abstract

In this paper we present an efficient solution, based on machine learning, to the problem of obtaining the phase of a set of interferograms. The algorithm learns the function from an interferogram, given as a gray-scale image, to a vector of aberration coefficients that form the Seidel aberration representation of the interferogram. The algorithm uses principal component analysis to reduce the dimensionality of the task and applies a modified version to the locally-weighted regression algorithm to find aberration coefficients. The method is faster than others previously presented in the literature, and is also noise-insensitive. We tested our algorithm using a large set of simulated interferograms, and our experiments show very accurate results using both noiseless and noisy interferograms.

Key Words:
active learning, instance-based learning, locally-weighted regression

1. Introduction

Interferometry is a laboratory technique very commonly used to test the quality of optical systems. To perform interferometry, two beams, one passing through a reference surface and the other passing through the test surface, are combined an made to interfere, which results in a pattern, called interferogram, that characterizes the quality of the test surface. A schematic diagram of a simple interferometer is shown in figure 1. Experienced technicians can diagnose the flaws of the test surface by careful analysis of the interferogram, however, this is a time consuming task, and when there is a need to analyze more than a few interferograms, it becomes impractical. Thus, there is a need for techniques to automate this process.

The problem of automatically characterizing an interferogram has received recent attention in the literature. This is a difficult problem, and traditional optimization schemes based on the least-squares method often provide inconclusive results, specially in the presence of noisy data [1, 2]. For this reason, non-traditional optimization schemes, such as evolutionary algorithms [3, 4], have been proposed to solve this problem [5]. While evolutionary algorithms provide very accurate results in the case of both noiseless and noisy data, their running time is high, taking several minutes to analyze a single interferogram. Clearly, if we need to analyze a large number of interferograms, this approach becomes unfeasible.

In this paper we propose an efficient algorithm to find the phase of a set of interferograms. We use a version of the well-known locally weighted regression algorithm [6], modified to automatically incorporate new available data in regions of the parameter space where the available data are too sparse, or the target function is too complex, to generate accurate predictions. To further reduce the running time, we use a principal component analysis preprocessing stage to compress the high-dimensional interferograms into a more manageable size with minimal loss of information.

The organization of the remainder of this paper is as follows: In Section 2 we describe the procedure we used to generate a set of noisy simulated interferograms. Section 3 presents the methods we used, including principal component analysis and locally-weighted regression. Section 4 gives a detailed description of our algorithm. In Section 5 the main results are shown, and Section 6 presents conclusions and suggest directions for future work

2. Interferogram Simulation

To obtain the simulated interferograms we use Kingslake's formulation, where the intensity in the interferogrametric image is given by

$$I(x,y) = A + B\,cos(\frac{2\pi}{\lambda}\,W(x,y)) + N(x,y)$$

where λ is the wavelength of the light source, N is the noise which we represented as a random number obtained from a normal distribution with zero mean, and $W(x,y)$ is the optical path difference (OPD) between the reference and test surfaces, and it is rep-

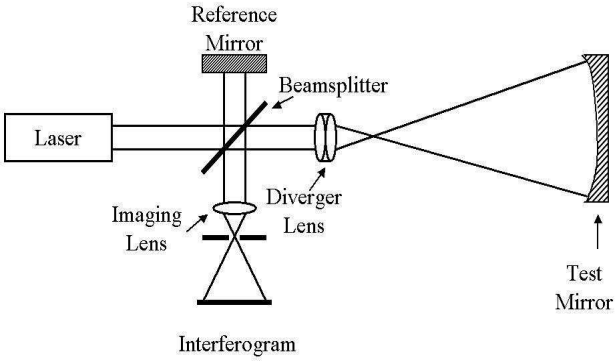


Figure 1. A simple interferometer

resented by a polynomial using the Seidel aberration formulation:

$$W(x,y) = A(x^2+y^2)^2 + By(x^2+y^2) + C(x^2+3y^2)$$
$$+D(x^2+y^2) + Ey + Fx$$

where A is the spherical aberration coefficient, B, C and D are the comma coefficient, astigmatism and defocusing coefficients, respectively, E is the tilt about the y axis, and F is the tilt about the x axis.

Clearly, it is easy to obtain an interferogram I given the vector of aberration coefficients $v = [A, B, C, D, E, F]$. However, we are interested in the inverse problem, that is, obtaining the vector of aberration coefficients from the corresponding interferogram, which, as mentioned before, is a very difficult optimization problem. Section 4 will describe our proposed solution to this problem.

3. The Methods

3.1 Principal Component Analysis

The formulation of standard PCA is as follows. Consider a set of M objects $O_1, O_2, \ldots, O_M$, where the mean object of the set is defined by

$$X = \frac{1}{M}\sum_{n=1}^{M} O_n \quad (1)$$

Each object differs from the mean by the vector

$$\theta_i = O_i - X \quad (2)$$

Principal component analysis seeks a set of M orthogonal vectors v and their associated eigenvalues k which best describes the distribution of the data. The vectors v and scalars k are the eigenvectors and eigenvalues, respectively, of the covariance matrix

$$C = \sum_{i=1}^{M}\sum_{n=1}^{M} \theta_n \theta_n^T = AA^T \quad (3)$$

where the matrix $A = [\theta_1, \theta_2, \ldots, \theta_M]$

The associated eigenvalues allow us to rank the eigenvectors according their usefulness in characterizing the variation among the objects.

Then, this module takes as input the set of interferograms (or images), and finds its principal components (PCs). Then the interferograms are projected onto the space defined by the first 30 principal components, which were found to account for about 90% of the variance in the set, and the magnitudes of these projections are used as attributes for the next stage in the system. That is, the projection *Proj* is given by

$$Proj = PCs^T(T - M) \quad (4)$$

where T is the training set of images and M is the mean image.

3.2 Locally-Weighted Regression

Locally-weighted regression belongs to the family of instance-based learning algorithms. In contrast to most other learning algorithms, which use their training examples to construct explicit global representations of the target function, instance-based learning algorithms simply store some or all of the training examples and postpone any generalization effort until a new instance must be classified. They can thus build query-specific local models, which attempt to fit the training examples only in a region around the query point. In this work we use a linear model around the query point to approximate the target function.

Given a query point $\mathbf{x}_q$, to predict its output parameters $\mathbf{y}_q$, we assign to each example in the training set a weight given by the inverse of the distance from the training point to the query point: $w_i = \frac{1}{|\mathbf{x}_q - \mathbf{x}_i|}$. Let W, the weight matrix, be a diagonal matrix with entries $w_1, \ldots, w_n$. Let X be a matrix whose rows are the vectors $\mathbf{x}_1, \ldots, \mathbf{x}_n$, the input parameters of the examples in the training set, with the addition of a "1" in the last column. Let Y be a matrix whose rows are the vectors $\mathbf{y}_1, \ldots, \mathbf{y}_n$, the output parameters of the examples in the training set. Then the weighted training data are given by $Z = WX$ and the weighted target function is $V = WY$. Then we use the estimator for the target function $\mathbf{y}_q = \mathbf{x}_q^T(Z^TZ)^{-1}Z^TV$.

Thus, locally weighted linear regression (LWLR) is very similar to least-squares linear regression, except that the error terms used to derive the best linear approximation are weighted by the inverse of their distance to the query point. Intuitively, this yields much

more accurate results than standard linear regression because the assumption that the target function is linear will not hold in general, but is a very good approximation when only a small neighborhood is considered.

I_s is a matrix whose rows are vectors of interferogram images
A_s is the vector of aberration coefficients
S is the training set, given by the tuple $[I_s, A_s]$
T is the test set of interferogram images

1. While $T \neq \oslash$ do:
   - Build C, the classifier, using S and LWLR
   - Use C to predict the aberration coefficients vector V for the test interferograms
   - Compute I_t, the interferograms that generated the predicted aberrations in V and measure similarity with T
     - Let N be the interferograms for which the similarity was greater than some threshold, do:
       $T = T - N$
       $I_t = I_t - N$
     - Add I_t together with their corresponding aberrations vector to the training set:
       $S = S \cup [I_t, V_t]$
2. End

Table 1. The optimization algorithm

4. The Optimization Algorithm

Normally, machine learning algorithms are passive in the sense that they just use the training data that are provided. Alternatively, an algorithm can be active, selecting the examples that are deemed most useful to be added to the training set. Moreover, the algorithm itself can produce those examples that are going to augment the training set. This can be more easily explained if we consider how instance based learning algorithms perform prediction tasks. What these algorithms do to predict the target function value for a new example is to select from the training set the most similar points to the new example and then, by some average metric, output the target value for the query. In the case of interferogram analysis, we can reverse the process and obtain a new instance based on the algorithm's output and measure how similar this new instance is to the query one (that is, we can easily obtain a new interfergram from the predicted value of the vector of aberration coefficients $v = [A, B, C, D, E, F]$). If the difference between the newly obtained interferogram and the one under test is high, we can add this new instance to the training set and allow the query instance to remain in the test set until the algorithm's output is considered similar enough. By doing so, the algorithm is able to use new instances that are presumably closer neighbors to the query point than the original training examples. See Table 1 for an outline of our algorithm.

What we are trying to do with this approach is to overcome one disadvantage of instance based-learning algorithms. When the query point lies too far from the training examples, it's difficult for the algorithm to output a target value accurate enough. However, it is likely that whatever the predicted value is, it will lie in the vicinity of the query point, presumably nearer than any of the training examples. By adding the new instance that was created based on the algorithms output, and repeating this step several times, eventually the algorithm will be provided with the needed examples to output a very accurate value.

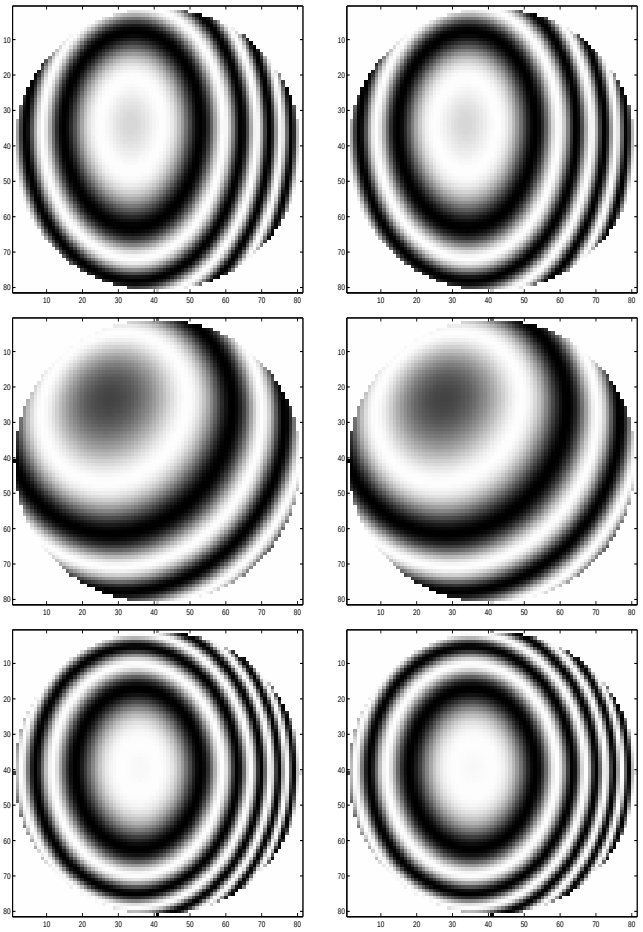
In the problem we are trying to solve here, interferogram analysis, the algorithm tries to predict the aberration coefficients for a given interferogram. Once the algorithm has predicted these aberration coefficients we can easily compute the corresponding interferogram. By comparing the images of the two interferograms we can determine if this prediction is good enough, if this is not the case, then the new generated interferogram is added to the training set while the test one remains in the test set.

Coefficients	**Mean Absolute Error**	**Standard Deviation**
A	3.9286e-003	3.5283e-004
B	4.5149e-003	6.8457e-004
C	1.5848e-003	2.0088e-004
D	2.6554e-003	2.3317e-004
E	5.4078e-004	1.2607e-004
F	1.0868e-003	9.6457e-005

Table 2. Mean Absolute Errors for Simulated Interferograms

5. Experimental Results

In this section we describe the experiments performed with our optimization algorithm applied to the problem of predicting the vector of aberration coefficients. First, we generated three thousand aberration vectors together with their corresponding interferograms. Then we randomly divided the data into two equally sized subgroups. One group was used for training and



a) Simulated interferograms

b) Predicted interferograms

Figure 2. Column (a) Shows three generated interferograms used for testing while column (b) shows the corresponding interferograms obtained from the predicted aberration coefficients.

the other was considered the test set. We repeated this procedure ten times, and the overall average are the results presented here. Table 2 shows averaged mean absolute errors for each aberration coefficient together with standard deviation. Figure 2 shows three pairs of interferograms, column a) corresponds to the original simulated interferograms, while column b) shows the interferograms that generated the aberration coefficients predicted by our optimization algorithm. As can be seen, the interferograms are practically identical.

As the experimental results show, our method is very accurate with the simulated interferograms. However, real data always pose the challenge of managing noise. In order to evaluate the noise sensibility of our method, we performed experiments on interferograms with simulated noise. In this experiment we also performed ten runs of the procedure described previously,

Coefficients	**Mean Absolute Error**	**Standard Deviation**
A	3.9652e-002	1.4999e-003
B	4.2934e-002	1.8648e-003
C	1.7704e-002	5.2013e-004
D	3.3045e-002	9.5258e-004
E	7.0914e-003	4.1191e-004
F	1.6100e-002	7.3746e-004

Table 3. Mean Absolute Errors for Noisy Simulated Interferograms

the only difference being that we added noise to the test interferograms. The noise is simulated as a Gaussian error, depending only on a mean and a standard deviation. Table 3 shows errors in aberration coefficients. In Figure 3 we can see a visual comparison between noisy interferograms and the interferograms obtained from the predicted aberrations. It can be seen that our method's performance was not damaged by the noise in the test data.

6. Conclusions

In this paper we have presented an efficient method to solve the problem, given a large set of interferograms, finding their corresponding vectors of aberration coefficients. The method yields very accurate results, even in the presence of noise, and also, it is significantly faster than other methods introduced earlier. One important feature of our method is the ability of extending the training set automatically, in order to increase prediction accuracy. Present and future work includes:

- Testing the method using real interferograms
- Extending the algorithm to handle higher-order aberrations
- Testing the applicability of the methods to other optimization problems in optics.

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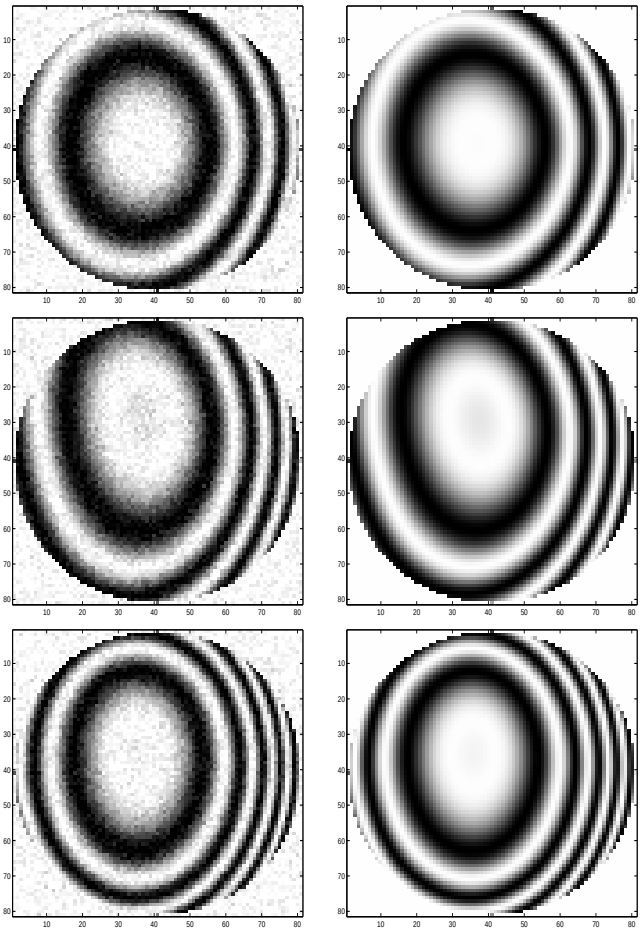

a) Simulated noisy interferograms

b) Predicted interferograms

Figure 3. Column (a) Shows three of the 3,000 noisy interferograms used for testing while column (b) shows the corresponding interferograms obtained from the predicted aberration coefficients.

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