Two-Dimensional Fitting of Brightness Profiles in Galaxy Images with a Hybrid Algorithm

Juan Carlos Gomez, Olac Fuentes, and Ivanio Puerari

Instituto Nacional de Astrofísica Óptica y Electrónica
Luis Enrique Erro # 1, Tonantzintla, Puebla, 72840, México
{jcgc,fuentes,puerari}@inaoep.mx

Abstract. Fitting brightness profiles of galaxies in one dimension is frequently done because it suffices for some applications and is simple to implement, but many studies now resort to two-dimensional fitting, because many well-resolved, nearby galaxies are often poorly fitted by standard one-dimensional models. For the fitting we use a model based on de Vaucoleurs and exponential functions that is represented as a set of concentric generalized ellipses that fit the brightness profile of the image. In the end, we have an artificial image that represents the light distribution in the real image, then we make a comparison between such artificial image and the original to measure how close the model is to the real image. The problem can be seen as an optimization problem because we need to minimize the difference between the original optical image and the model, following a normalized Euclidean distance.
In this work we present a solution to such problem from a point of view of optimization using a hybrid algorithm, based on the combination of Evolution Strategies and the Quasi-Newton method. Results presented here show that the hybrid algorithm is very well suited to solve the problem, because it can find the solutions in almost all the cases and with a relatively low cost.

1 Introduction

Galaxies span a wide range of morphology and luminosity, and a very useful way to quantify them is to fit their light distribution. Fitting profiles for galaxies in one dimension is frequently done because it suffices for some applications and is simple to implement [5], but many studies now resort to two-dimensional fitting, because many well-resolved, nearby galaxies are often poorly fitted by standard one-dimensional models. Although empirical techniques for galaxy fitting and decomposition have led to a number of notable advances in understanding galaxy formation and evolution, many galaxies with complex isophotes, ellipticity changes, and position angle twists can be modelled accurately in two dimensions. We illustrated this by 5 examples, which include elliptical and spiral galaxies displaying various levels of complexities. In one dimension, the galaxy bulge and disk may appear to merge smoothly, which causes non-uniqueness in the decompositions, while in two dimensions ishophote twists and ellipticity change provide additional constraints to break those degeneracies.

R. Khosla et al. (Eds.): KES 2005, LNAI 3682, pp. 179–185, 2005.

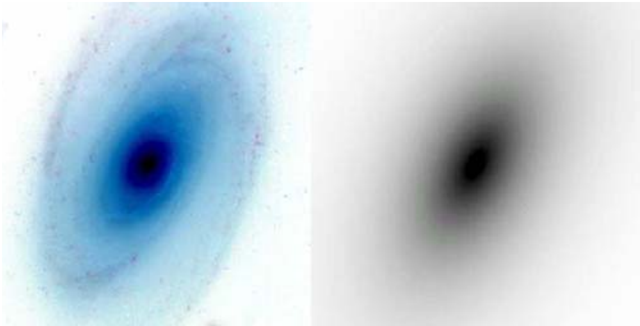


Fig. 1. Galaxy image and its modelled brightness profile

The algorithms we are using here are: Evolution Strategies (ES) [1, 6], Quasi-Newton (QN) [2, 3] and a hybrid algorithm, merging both of them. The hybrid algorithm takes advantage of the main features from the two previous algorithms: the global exploration of ES and the fast convergence of QN when the search is near the solution.

The rest of the paper is structured as follows: in Section 2 a brief description of theory of brightness profiling and a description of the problem are presented, the optimization methods are shown in Section 3, Section 4 includes the general description of the implementation, results are presented in Section 5 and Section 6 includes conclusions and future work.

2 Brightness Profile

Surface brightness in a galaxy is literally defined as how much light the galaxy emits[5], and luminosity is defined as the total energy received per unit of area per unit of time. Then, the surface brightness of an astronomical source is the ratio of the source's luminosity F and the solid angle (Ω) subtended by the source:

$$B = \frac{F}{\Omega} \tag{1}$$

The surface brightness profile in an elliptical galaxy follows the de Vaucoulers Law $r^{1/4}$ [4]:

$$I_b = I_e \exp\left[-7.67\left[\left(\frac{r^{1/4}}{r_e}\right) - 1\right]\right] \tag{2}$$

where r is the distance from the galactic center, r_e is the mean ratio of the galaxy brightness (the radius where half of the total brightness lies), and I_e is the surface brightness for $r = r_e$.

Also, the surface brightness profile for a disc galaxy has an exponential distribution:

$$I_d = I_0 \exp\left(-\frac{r}{r_d}\right) \tag{3}$$

where I_0 is the central surface brightness and r_d is the radial scale length.

Finally, surface brightness distribution in elliptical and spiral galaxies can be described as the sum of equations (2) and (3), which is an approximation of the profile using concentric ellipses [5].

$$I_d = I_b + I_d \tag{4}$$

In fact, it is not expected that equations (2) and (3) fit all the profiles measured in the radial range of the galaxy, because sometimes sky substraction errors in external regions of galaxy can distort the profile. An example of a galaxy image and its corresponding generated brightness profile using the previous equations are shown in Figure 1.

The problem of two-dimensional fitting can be described in brief as follows: given an image, taken from photometric observations, of a spiral or elliptical galaxy, an exploration of search space will be done to estimate a set of parameters that define the surface brightness profile of the galaxy. Parameters to be determined by the algorithms are: r_e, mean ratio of the galaxy brightness; I_e, surface brightness in $r = r_e$; I_0, central surface brightness; r_d, radial scale length and two angles i_1 and i_2, which are the rotation angles about the x and z axes, with the x axis being horizontal and the z axis pointing towards the observer.

3 Optimization Methods

3.1 Evolution Strategies

Evolution Strategies (ES) [1, 6] is concerned with finding the global minimum of a function with a large number of variables in a continuous space. The algorithm starts by choosing k, $\{\mathbf{q}_1, \mathbf{q}_2, \cdots, \mathbf{q}_k\}$ individuals, each characterized by an object parameter vector $\mathbf{q}$ and a corresponding strategy parameter vector $\mathbf{s}$:

$$\mathbf{q}_i = \langle q_{1,i}, q_{2,i}, \cdots, q_{L,i}\rangle \; i = 1, \cdots, k$$
$$\mathbf{s}_i = \langle \sigma_{1,i}, \sigma_{2,i}, \cdots, \sigma_{L,i}\rangle \; i = 1, \cdots, k$$

In the first generation the elements of the $\mathbf{q}$ and $\mathbf{s}$ vectors can be chosen totally at random. Each of the k individuals must be evaluated according to a fitness function. The fitness function is what we need to minimize (or maximize, depending on the point of view), and also is called *target function*.

The next step is to produce a new population applying the genetic operators cross-over and mutation. For cross-over, two individuals (parents) are chosen at random, and then we create two new individuals (offspring) by combining the parameters of the two parents.

Mutation is applied to the individuals resulting from the cross-over operation. Each element of the new individual is calculated from the old individual using the simple operation: $\mathbf{q}_{j,\mathrm{mut}} = \mathbf{q}_j + N(0, \sigma_j)$, where $N(0, \sigma_j)$ is a random number obtained from a normal distribution with zero mean and standard deviation σ_j, which is given by the strategy parameter vector. The process of cross-over and mutation is repeated until the population converges to a suitable solution.

3.2 Quasi-Newton

To solve a system of non-linear equations Newton's method has to estimate the Jacobian in each iteration, which implies computing partial derivatives; that results in a very high computational cost. To avoid that complexity, Quasi-Newton (QN) method [2, 3] substitutes the Jacobian Matrix in Newton's method with an approximate matrix that is recalculated in each iteration.

In the beginning, the Jacobian $J(x_1)$ is substituted by a matrix A_1 in Newton's method:

$$A_1 = J(x_0) + \frac{[f(x_1) - f(x_0) - J(x_0)(x_1 - x_0)]\,(x_1 - x_0)^t}{\|x_1 - x_0\|^2} \tag{5}$$

Then, this matrix is used to calculate x_2:

$$x_2 = x_1 - A_1^{-1} f(x_1) \tag{6}$$

Quasi-Newton works with the two previous equations, substituting the corresponding matrix for each iteration. It is possible to calculate the inverse matrix in (9) with the following equation, avoiding the inversion process in each iteration:

$$A_i^{-1} = A_{i-1}^{-1} + \frac{(s_i - A_{i-1}^{-1} y_i) s_i^t A_{i-1}^{-1}}{s_i^t A_{i-1}^{-1} y_i} \tag{7}$$

where $y_i = f(x_i) - f(x_{i-1})$ and $s_i = x_i - x_{i-1}$.

The previous equation uses only matrix multiplication, then the total number of operations in the whole process has a complexity $O(n^2)$.

3.3 Hybrid Algorithm

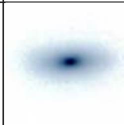
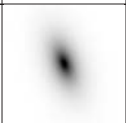
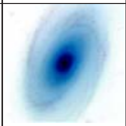
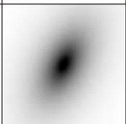
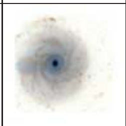
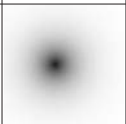
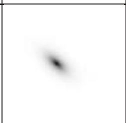
The hybrid algorithm used here is ES+QN. This hybrid algorithm is implemented using ES as a global method to identify promising regions in the search space, and then, once the region is located, we switch to a QN as local method to refine the best solution found by ES.

The employed metric to determine where the interesting region lies and when to change between algorithms is based on progress of the global algorithm, that is, if the ES algorithm has found a stable region, it is a good sign indicating that the optimum is near. So when the algorithm ES has not experimented an improvement of 10% in its best individual during 10 generations, we switch to the QN algorithm.

4 Implementation

ES were implemented using a population of 10 individuals and was evolved until the target function reached a value greater than 0.95. QN was implemented using the function FMINUNC included in the Optimization Toolbox of Matlab, using all the default parameters.

Table 1. Results for a set of galaxy images

Galaxy	Original Image	Best Model	Type	Algorithm	Function Evaluations	$\frac{1}{1+\frac{\|A-B\|^2}{B+\epsilon}}$
NGC2768			Elliptical	QN	N/C	N/C
				ES	5139	0.9824
				Hybrid	**2970**	0.9824
NGC2903			Spiral	QN	N/C	N/C
				ES	**3348**	0.9719
				Hybrid	N/C	N/C
NGC3031			Spiral	QN	N/C	N/C
				ES	5737	0.9616
				Hybrid	**4716**	0.9664
NGC3344			Spiral	QN	**159**	0.9514
				ES	2430	0.9514
				Hybrid	1588	0.9514
NGC4564			Elliptical	QN	**302**	0.9918
				ES	3671	0.9917
				Hybrid	693	0.9918

In the beginning we have an image of a spiral or elliptical galaxy. By convention we chose working with 256x256 images in grey scale. In the image we first determine the galactic center by finding the brightest pixels in a box of 10x10 pixels, in this box we take the central pixel and use its coordinates as the center of the galaxy in the image.

The process starts by generating randomly a vector (or set of vectors in the case of ES) of numbers which represents the set of brightness parameters described in Section 2: $[i_{1,j}, i_{2,j}, I_{e,j}, r_{e,j}, I_{0,j}, r_{d,j}]$

The parameter vector is used as input for a program which creates an artificial image that represents the light distribution in the galaxy, following equations (2) and (3). The obtained image is also 256x256 pixels in grey scale. Then, the artificial image is evaluated with the following target function:

$$f = \frac{1}{1 + \frac{\|A-B\|^2}{B+\epsilon}} \tag{8}$$

where A is the artificial image, B is the original optical image and ϵ is a very small constant to prevent division by 0. The target function represents the similarity between both images, and its value range is between 0 and 1, with 1 as a perfect match and 0 as totally different images. At the end, the simulated image that maximizes this equation is the one that was produced by the set of brightness parameters we are looking for. We established empirically that with a minimum value of 0.95 for the target function, the difference between images is almost imperceptible.

If the target function has not reached the tolerance, then the next step is to make a modification to the parameter vector. This modification is done according to the optimization method (ES or QN), following particular features of each technique, as seen in Section 3. Once the modification is done, the process is repeated again.

The number of times the process can be repeated is determined in different ways for each algorithm: for ES we established that the process stops if during 10 generations the target function has not been enhanced in a value greater than 0.3, and Matlab determines the total number of evaluations QN can do.

In the case of the hybrid algorithm, we start with ES and evolved the population until the target function has a value greater than 0.9, or if during 10 generations the target function has not been improved in a value greater than 0.1, then we change to QN, with the best individual reached until that moment as the starting point for the local algorithm.

5 Results

Table 1 shows the results obtained after applying the algorithms to a set of galaxy images. The first column indicates the name of the galaxy, the second and third columns show the original and best model reached (by one of the three algorithms) respectively, the fourth one the kind of algorithm used, the fifth one the total number function calls needed by the algorithm to reach convergence and the last one shows the value for the target function for the maximum found (in a normalized quantity between 0 and 1, with 1 as a perfect match).

In the fifth column the smallest number of function calls is marked with bold text, which indicates the algorithm with the best behavior in each example. We choose the number of function calls as the measure of efficiency because, in most cases, all the algorithms have very similar accuracies at the end.

From the table we can see that the hybrid algorithm has a very good behavior, as was expected, it found four of five solutions, and the cost is lower than ES (and in some cases similar to QN). QN is the algorithm with the most deficient behavior, since it presents difficulties to reach convergence in three of the five examples, but, on the other hand, when the algorithm was able to find a solution it did so with only a few iterations. Also from the figure we see that ES is the algorithm with the most stable behavior, it was able to find good solutions for all the cases, but the cost to reach an acceptable model is very high in comparison with QN and the hybrid algorithm.

6 Conclusions

In this work we have solved the problem of two-dimensional fitting of brightness profiles for spiral and elliptical galaxies using a hybrid algorithm, based on a global optimization algorithm and a local optimization traditional algorithm. The hybrid method was compared with the two optimization techniques separately. The hybrid algorithm achieved the best results considering the total number of iterations and the number of solutions found. QN was the worst because it was not able to find solutions in most of the cases, as was expected, because the problem of fitting profiles is a complex problem with real noise, and this kind of algorithm are not very well suited to work with this. ES is the most reliable algorithm to find a solution, but the cost of finding a model can be very high. Thus, it is possible to conclude that the hybrid algorithm outperforms QN and ES in most of the examples.

References

1. Back, T., Hoffmeister, F., Schwefel, H. P., A Survey of Evolution Strategies, Proceedings of the Fourth International Conference on Genetic Algorithms. (1991), 2-9
2. Coleman, T. F., Li, Y., An Interior, Trust Region Approach for Nonlinear Minimization Subject to Bounds, SIAM Journal on Optimization. (1996), 6, 418-445.
3. Coleman, T. F., Li, Y., 1994, On the Convergence of Reflective Newton Methods for Large-Scale Nonlinear Minimization Subject to Bounds, Mathematical Programming. (1994), 67, 2, 189-224.
4. de Vaucouleurs, G., Recherches sur les Nebuleuses Extragalactiques, Ann. d'Astrophys. (1948), 11, 247
5. Karttunen, H., Fundamental Astronomy, Springer-Verlag. (1996)
6. Rechenberg, I., Evolutionsstrategie: Optimierung technischer Systeme nach Prinzipien der biologischen Evolution, Stuttgart: Fromman-Holzboog. (1973)