Dimension Compactification – a Possible Explanation for Superclusters and for Empirical Evidence Usually Interpreted as Dark Matter

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Abstract

According to modern quantum physics, at the microlevel, the dimension of space-time is ≥ 11; we only observe 4 dimensions because the others are compactified: the size along each of the other dimensions is much smaller than the macroscale. There is no universally accepted explanation of why exactly 4 dimensions remain at the microscopic level. A natural suggestion is: maybe there is no fundamental reason why exactly 4 dimensions should remain, maybe when we go to even larger scales, some of the 4 dimensions will be compactified as well? In this paper, we explore the consequences of the compactification suggestion, and we show that – on the qualitative level – these consequences have actually been already observed: as superclusters and as evidence that is usually interpreted as justifying dark matter. Thus, we get a new possible explanation of both superclusters and dark matter evidence – via dimension compactification.

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Main idea. According to modern quantum physics, at the microlevel, space-time has at least 11 dimensions; we only observe 4 dimensions in our macroobservations because the rest are *compactified:* the size along each of the remaining directions is so small that on macrolevel, these dimensions can be safely ignored; see, e.g., [Green et al. 1988, Polchinski 1998].

There is no universally accepted explanation of why exactly 4 dimensions remain at the microscopic level. A natural suggestion is: maybe there is no fundamental reason why exactly 4 dimensions should remain. Maybe when we go to even larger scales, some of the 4 dimensions will be compactified as well?

Could one rigorously argue for "compactification" when these effects must occur over the accessible, kiloparsec and larger, scales? In modern physics, indeed, compactification is related to quantum-size distances, but there is nothing inherently quantum in the compactification idea. Indeed, the very first paper that proposed compactification – the 1938 paper by A. Einstein and P. Bergmann [Einstein and Bergmann 1938] – described it as simply the extra dimension being a circle, so that the entire space-time looks like a thin cylinder whose width is negligible if we operate at large enough scales.

In this paper, we explore the consequences of the compactification suggestion, and we show that – on the qualitative level – these consequences have actually been already observed: as superclusters and as evidence that is usually interpreted as justifying dark matter. Thus, we get a new possible explanation of both superclusters and dark matter evidence – via dimension compactification.

Geometric consequences of the main idea. If our idea is correct, then, as we increase the scale, we will observe how a 3D picture is replaced by a 2D and then by a 1D one. This is exactly what we observe: while at a macrolevel, we see a uniform distribution of galaxies in a 3D space, at a larger-scale level, galaxies start forming *superclusters* – long and thin strands of clusters and galaxies; see, e.g., [Fairall 1998]. Superclusters are either close to a 2D shape (as the "Great Wall" discovered in the 1980s) or close to 1D.

Towards physical consequences of the main idea. How does the change in dimension affect physics? In non-relativistic approximation, the gravitation potential φ is related to the mass density ρ by the Laplace equation $\triangle\varphi = \rho$. In the 3D space, this leads to Newton's potential $\sim 1/r$, with the force

$$F(r) = \frac{G_0 \cdot m_1 \cdot m_2}{r^2};$$

in a 2D space, we get potential $\sim \log(r)$, with the force

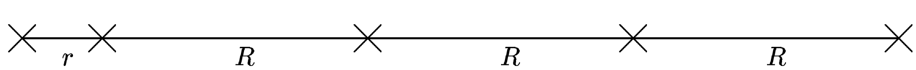
$$F(r) = \frac{G_1 \cdot m_1 \cdot m_2}{r}$$

for some constant G_1. For intermediate scales, it is reasonable to consider a combination of these two forces:

$$F(r) = \frac{G_0 \cdot m_1 \cdot m_2}{r^2} + \frac{G_1 \cdot m_1 \cdot m_2}{r}. \tag{1}$$

In the Appendix, we explain, on the qualitative level, why such combination naturally follows from the above compactification idea.

Let us consider the simplest possible compactification (along the lines of the original Einstein-Bergmann paper), where the space-time is wrapped as a cylinder of circumference R along one of the coordinates. What will the gravitational potential look like in this simple model? The easiest way to solve the corresponding Newton's equation in this cylinder is to "unwrap" the cylinder into a full space. After this "unwrapping", each particle in a cylindrical space (in particular, each source of gravitation) is represented as infinitely many different bodies at distances R, $2R$, etc., from each other. For further simplicity, let us consider the potential force between the two bodies on the wrapping line at a distance r from each other. The effect of the second body on the first one in cylindrical space is equivalent to the joint effect of multiple copies of the second body in the unwrapped space:



The resulting gravitational potential of a unit mass can be described as a sum of potentials corresponding to all these copies, i.e.,

$$\varphi(r) = \frac{1}{r} + \frac{1}{r+R} + \frac{1}{r+2R} + \ldots + \frac{1}{r+k\cdot R} + \ldots \tag{2}$$

From the purely mathematical viewpoint, this sum is infinite. From the physical viewpoint, however, the actual potential is not infinite: due to relativistic effects, at the current moment of time t_0, the influence of a source at a distance $d = r + k \cdot R$ is determined by this source's location at a time $t_0 - d/c$ (where c is the speed of light). Thus, we only need to add the terms for which d/c is smaller than the age of the Universe. As a result, we can ignore the slowly increasing infiniteness of the sum when $k \to \infty$.

How can we estimate this potential? The formula (2) has the form

$$f(0) + f(1) + \ldots + f(k) + \ldots,$$

where we denoted $f(x) \stackrel{\text{def}}{=} 1/(r + x \cdot R)$. It is difficult to get an analytical expression for the exact sum, but we can use the fact that this sum is an integral sum for an integral $\int_0^\infty f(x)\,dx$; this integral has an analytical expression – it is const $-$ $(1/R)\cdot\ln(r)$. In this approximation

$$\int_0^\infty f(x)\,dx \approx f(0) + f(1) + \ldots + f(k) + \ldots, \tag{3}$$

we used the fact that

$$\int_0^\infty f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \ldots + \int_k^{k+1} f(x)\,dx + \ldots,$$

and we approximated each term $\int_k^{k+1} f(x)\,dx$ by $f(k)$. This approximation is equivalent to approximating the function $f(x)$ on the interval $[k, k+1]$ by its value $f(k)$ at the left endpoint of this interval – i.e., by the first term in the Taylor expansion of the function $f(x)$ around the point k. A natural next approximation is when instead of only taking the first term, we consider the first *two* terms in this Taylor expansion, i.e., when on each interval $[k, k+1]$, we approximate the function $f(x)$ by a formula $f(k) + f'(k)\cdot(x - k)$. Under this approximation,

$$\int_k^{k+1} f(x)\,dx \approx f(k) + \frac{1}{2}\cdot f'(k),$$

and therefore,

$$\int_0^\infty f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \ldots + \int_k^{k+1} f(x)\,dx + \ldots =$$

$$(f(0) + f(1) + \ldots + f(k) + \ldots) + \frac{1}{2}\cdot(f'(0) + f'(1) + \ldots + f'(k) + \ldots). \quad (4)$$

The second term in the right-hand side can be (similarly to the formula (3)) approximated by

$$f'(0) + f'(1) + \ldots + f'(k) + \ldots \approx \text{const} + \int_0^\infty f'(x)\,dx - \frac{1}{2}\cdot f(0).$$

Thus, from the formula (4), we can conclude that

$$f(0) + f(1) + \ldots + f(k) + \ldots \approx \int_0^\infty f(x)\,dx + f(0).$$

In particular, for our function $f(x)$, we get

$$\varphi(r) = f(0) + f(1) + \ldots + f(k) + \ldots \approx \text{const} - \frac{1}{R}\cdot \ln(r) + \frac{1}{2}\cdot\frac{1}{r}.$$

Differentiating relative to r, we get the desired formula (1) for the gravitational force:

$$F(r) = -\frac{d\varphi(r)}{dr} = \frac{1}{R\cdot r} + \frac{1}{2r^2},$$

with $R = 2G_0/G_1$. According to estimates from [Kuhn and Krugliak 1987], we expect R to be between ≈ 10 and ≈ 30 kpc.

Physical consequences of the main idea. The force described by formula (1) is exactly the force that, according to Kuhn, Milgrom, et al. [Kuhn and Krugliak 1987, Milgrom 1983, Milgrom 2002, Sanders and McGaugh 2002], is empirically needed to describe the observations if we want to avoid dark matter. Indeed, in Newtonian mechanics, for any large-scale rotating gravitational system, if we know the rotation speed v at a distance r from the center, we can find the mass $M_g(r)$ inside the sphere of radius r by equating the acceleration v^2/r with the acceleration $G_0\cdot M_g(r)/r^2$

provided by the Newton's law. As a result, we get $M_g(r) = r \cdot v^2/G_0$. Alternatively, we can also count masses of different observed bodies and get $M_L(r)$ – the total mass of luminescent bodies. It turns out that $M_L(r) \ll M_g(r)$ – hence the traditional explanation that in addition to luminescent bodies, there is also "dark" (non-luminescent) matter.

An alternative explanation is not to introduce any new unknown type of matter – i.e., assume that $M_g(r) \approx M_L(r)$ – but rather change the expression for the force, or, equivalently, assume that the gravitational constant G_0 is not a constant but it may depend on r: $G_0 = G(r)$. Equating the acceleration v^2/r with the acceleration $G(r) \cdot M_L(r)/r^2$ provided by the new gravity law, we can determine $G(r)$ as $G(r) = v^2 \cdot r/M_L(r)$. Observation data show that $G(r) = G_0 + G_1 \cdot r$ for some constant G_1 – i.e., that the dependence of the gravity force on distance is described by the formula

$$F(r) = \frac{G(r) \cdot m_1 \cdot m_2}{r^2} = \frac{G_0 \cdot m_1 \cdot m_2}{r^2} + \frac{G_1 \cdot m_1 \cdot m_2}{r},$$

which is exactly what we deduced from our dimension compactification idea.

This idea has been proposed 20 years ago, and one of the reasons why it has not been universally accepted is that it was difficult to get a natural field theory explanation of this empirical law. We have just shown that such an empirical explanation comes naturally if we consider the possibility of dimension compactification.

This explanation is in line with Milgrom's own explanation; see, e.g., [Milgrom 2003].

As we go further up in scale, one more dimension starts compactifying, so we start getting a 1D space in which Laplace equation leads to potential $\varphi(r) \sim r$. Thus, at a large-scale level, we should have a term proportional to r added to the normal gravity potential formulas. This additional term is exactly what is add when we take a cosmological constant Λ into consideration.

Observable predictions of our new idea. A possible observable consequence of the additional term $\varphi(r) \sim r$ is that it leads to an additional constant term in the gravitational force and therefore, to a formula $G(r) = G_0 + G_1 \cdot r + G_2 \cdot r^2$.

Thus, if the empirical dependence of $G(r)$ on r turns out to be not exactly linear but rather slightly quadratic, it will be a strong argument in favor of our compactification idea.

Natural open questions. In this paper, we simply formulate the idea and explain why we believe this idea to be prospective. Many related questions are still open:

- how to concoct a geometry that accomplishes "compactification" – what would a metric look like that transitions from large to small scales?
- could we solve for the force law in such a geometry with some rigor?
- will such a profound geometrical perturbation as the one we propose have other consequences beyond a force law change? perhaps not, but it is worth investigating.

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