

OPTIMAL BIT ALLOCATION FOR MAXIMUM ABSOLUTE ERROR DISTORTION IN THE APPLICATION OF JPEG2000 PART 2

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ABSTRACT

This paper proposes a strategy to deal with bit rate allocation for the optimization of Maximum Absolute Error (MAE) or l-infinity distortion metric in 3-D data compression using JPEG2000. Part 2 of this standard has the capability to compress 3-D data by treating data as separate 2-D slices; these slices could be taken directly from the data or from the data after it has undergone a decorrelation transform (KLT) in one direction. To perform bit rate allocation we use a Mixed Model approximation to the MAE rate-distortion curve, that is used in an optimization algorithm. To solve the problem of MAE-related bit rate allocation in the KLT domain, we theoretically derive an upper bound for MAE based on the basis vectors of the KLT; we also develop an algorithm for optimizing this upper bound, and we illustrate how the minimization of this upper bound can decrease the actual MAE.

1. INTRODUCTION

In many application areas such as meteorological (Met) data processing, it is important to guarantee the same compression-decompression accuracy for all data points, i.e., in mathematical terms, it is important to control or minimize MAE. In this paper, we deal with optimal bit rate allocation for 3-D data compression using JPEG2000 [1] by treating data as separate 2-D slices that could be taken directly from the data or from the data that has undergone a decorrelation transformation in one direction [2]. Usually the *Karhunen – Loève Transform* (KLT) is used.

Recently, in [3] and [4] we proposed several approaches for this problem when the distortion metric is Mean Squared Error (MSE). In this paper, we use MAE as the distortion

metric. In particular, here we propose to use almost the same Mixed Model (MM) as in [3], [4] as a good approximation to a MAE rate distortion curve.

To solve the problem of bit rate allocation for the decorrelated 3-D data, we propose to use a derived theoretical upper bound for the total MAE error. Using a Lagrange multipliers approach, we solve the corresponding minimization problem and briefly illustrate with actual Met data.

2. OPTIMIZATION IN DATA DOMAIN (NO DECORRELATION)

In the *direct* optimization problem, we are given a target average bit rate for the collection of N slices as $R = 1/N \cdot \sum_{z=1}^N R_z$ and we want to find, among possible feasible bit rate combinations $\{R_1, \dots, R_N\}$ a specific choice (allocation) for which the Maximum Absolute Error (MAE) distortion attains the smallest possible value. In the *inverse* optimization problem, we want to find, among all bit rate allocations that guarantee a given MAE bound, the allocation $\{R_1, \dots, R_N\}$ for which the average bit rate is the smallest.

Let us first consider the simplest case, when no decorrelation is performed. For both direct and indirect optimization we assume that we know points on each slice's Rate Distortion Curve (RDC). These are values of $MAE_z(R_z)$ for various rates $R_z = R(t)$ for t from 1 to T_z (we either gather this information through actual compression experiments or by using a model). We propose (and justify) the following simple idea. The solution to the inverse problem, where MAE_0 is the given bound, is obtained by solving for each R_z in each of the following equations $MAE_z(R_z) = MAE_0$.

Let us assume that MAE_0 is an upper bound in an inequality. This means that for each layer z , we must select a bit rate R_z for which the corresponding $MAE_z(R_z)$ does not exceed the given bound MAE_0 . Our objective is to minimize the average bit rate. Assuming that smaller bit

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rates R_z produce larger distortions, to attain the smallest possible average bit rate, each slice's bit rate R_z is chosen to produce the maximum allowed value of MAE_0 . A similar argument can help us solve the direct optimization problem. The solution in this case is the set of bit rates that has the prescribed average and for every z , the equation $MAE_z(R_z) = MAE_0$ is satisfied. In this case, MAE_0 is obtained by searching through various values and checking that the constraint is satisfied.

3. RATE DISTORTION CURVES: THE MIXED MODEL APPROXIMATION

To generate experimental rate distortion curves, we select for each slice z several different increasing bit rates $R_z(1), R_z(2), \dots, R_z(T_z)$ in the acceptable range (from $RMIN_z$ to $RMAX_z$) and obtain the corresponding $MAE_z(R_z)$ at each point using a JPEG2000 coder and decoder.

To solve the direct and inverse optimization problems with respect to MAE we use both fully experimental RDCs (experimentally acquired pairs $(R_z, MAE_z(R_z))$) and rate-distortion curves generated with the use of the following Mixed Model (MM) (see [3], [4], [6] for motivation):

$$MAE(R) = \begin{cases} A \cdot \frac{1}{(R - R_0)^\alpha} & \text{if } R \leq \tilde{R}, \\ B \cdot 2^{-R} & \text{if } R > \tilde{R}. \end{cases} \quad (1)$$

The use of the MM requires for each slice z the determination of parameters: $A_z, R_{0z}, \alpha_z, B_z$, and \tilde{R}_z . The parameter B is computed as $B_z = MAE_z(R_H) \cdot 2^{R_H}$ for some high bit rate $R_H > \tilde{R}$.

To determine A_z and α_z , we consider the low-bitrate part of the curve. For reasonable accuracy, we need to consider $K > 2$ small rates R_{L1}, \dots, R_{LK} , for which we experimentally determine $MAE(R_{L1}), \dots, MAE(R_{LK})$. For meteorological data $K = 3$ works fine, however, for other types of data we may need to consider bigger values of K . We have devised a general approach corresponding to an arbitrary K , which essentially corresponds to a careful fitting of this model to fixed points on the curve (using a least-squares method).

If we choose low bit rates R_{L1}, \dots, R_{LK} such that they are significantly greater than R_{0z} , then we can use the simplified approximation

$$MAE_z(R) = A_z \cdot \frac{1}{R^{\alpha_z}}, \quad (2)$$

Taking logarithm of both parts of (2), we obtain

$$\log_2(MAE_z(R)) = \log_2(A_z) - \alpha_z \cdot \log_2(R), \quad (3)$$

For each R_i , the corresponding equation (3) must hold. In the left part of these equations we have experimental data.

The right part describes the assumed functional relationship, with unknown parameters A_z and α_z . For convenience, let us denote $\log_2(MAE_z(R_{L_i}))$ by y_i , $\log_2(A_z)$ by a , α_z by b , and $\log_2(R_{L_i})$ by x_i .

Then for each i , the expression $a + b \cdot x_i$ describes the predicted value by the model, and y_i is actual experimental value. We will use the least-squares method to minimize the measure of the misfit/error between the model and the data. Thus, we solve:

$$\sum_{i=1}^K (y_i - a - b \cdot x_i)^2 \rightarrow \min_{a,b} \quad (4)$$

by setting the derivatives with respect to a and b to zero.

By setting the derivative with respect to a to zero, dividing it by K and using notations

$$\bar{y} \stackrel{\text{def}}{=} \frac{y_1 + \dots + y_K}{K}, \bar{x} \stackrel{\text{def}}{=} \frac{x_1 + \dots + x_K}{K}, \quad (5)$$

we obtain

$$\bar{y} = a + b \cdot \bar{x}. \quad (6)$$

Similarly, by setting the derivative with respect to b to zero we can obtain

$$\bar{x}\bar{y} = a \cdot \bar{x} + b \cdot \bar{x}^2. \quad (7)$$

Solving for a and b , we obtain the following closed form expressions for α_z and A_z .

$$b = \alpha_z = \frac{\bar{x}\bar{y} - \bar{x} \cdot \bar{y}}{\bar{x}^2 - (\bar{x})^2}, \quad (8)$$

hence

$$A_z = 2 \left(\bar{y} - \frac{\bar{x} \cdot (\bar{x}\bar{y} - \bar{x} \cdot \bar{y})}{\bar{x}^2 - (\bar{x})^2} \right). \quad (9)$$

Once A_z and α_z are determined, we can use the maximum value of the slice I_z to compute R_{0z} as

$$R_{0z} = (A_z / \max(I_z))^{1/\alpha_z}.$$

The remaining parameter \tilde{R}_z can normally be determined by finding the intersection point between the first and second formulas in (1) (see Figure 1). However, in some cases we may not have intersection at all (this means that the curves coincide and we can use the first expression for all bit rates).

We can perform data domain bit rate allocation using either fully experimental RDCs or MM RDCs by finding the points where MAE_0 is achieved for all slices. The experimental RDCs are pre-processed to eliminate the experimental points that violate the monotonicity assumption. An actual MAE RDC is shown in Fig 1. The actual search utilizes the bisection algorithm to find the solution.

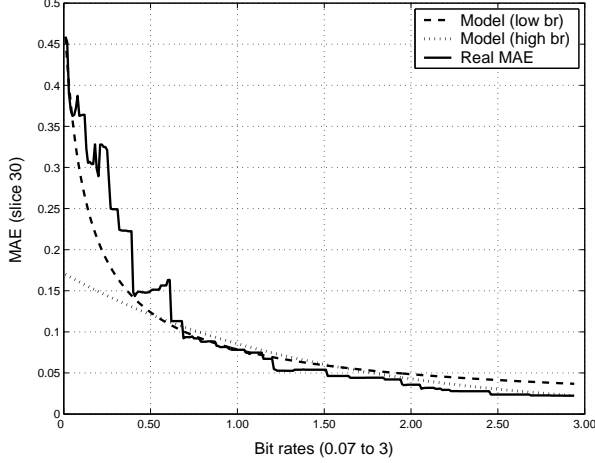


Fig. 1. T variable: experimental MAE rate distortion curves and MM components

For MM RDCs we can actually solve the corresponding equations and find two possible bit rates for each z . By analyzing the relative position of those rates w.r.t. \tilde{R} , we select one bit rate as the one fitting the overall model. The search (algorithmic implementation) for the direct optimization problem is also done using the bisection algorithm.

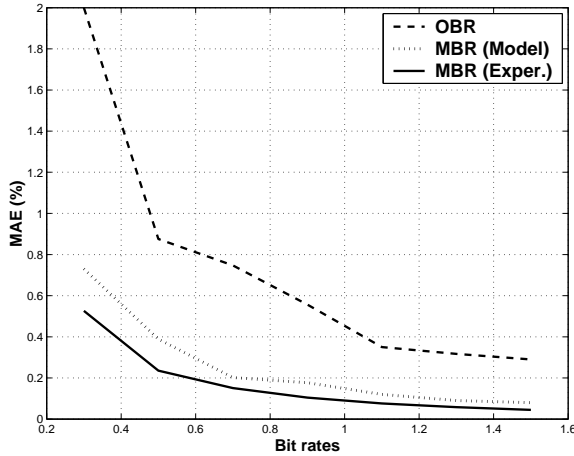


Fig. 2. U variable, direct problem, MAE for the different bit rate allocation approaches.

Figure 2 shows that the MM approach achieves results that are very close to those obtained using experimental RDCs, therefore in the future sections we use only MM as rate distortion curves. Note that we achieve improvement on the order of 3 times smaller MAE in comparison to one bit rate for all slices (OBR approach). An MAE below 1% is considered to be a good result for Meteorological applications.

4. BIT RATE ALLOCATION AFTER KLT PRE-PROCESSING

The presence of KLT-based pre-processing makes it more difficult to minimize MAE than MSE: indeed, while MSE is preserved under the KLT transform (due to Parseval's theorem), MAE is not preserved. It is therefore necessary, before we start minimizing MAE, to derive the explicit expressions for the guaranteed upper bound on MAE.

The use of KLT-based pre-processing means that we use the following representation of the original data:

$$\vec{I}(x, y) = \vec{I}_0 + a_1(x, y) \cdot \vec{e}_1 + \dots + a_N(x, y) \cdot \vec{e}_N; \quad (10)$$

where $\vec{I}(x, y)$ is the N -component vertical vector containing the values of the quantity I at different heights at the fixed point (x, y) , \vec{I}_0 is the average of all the vertical vectors, and $\vec{e}_1, \dots, \vec{e}_N$ are the eigenvectors of the covariance matrix characterizing the family of N -vector.

After compression/decompression, we get a degraded data vector $\vec{\tilde{I}}(x, y)$ of the form

$$\vec{\tilde{I}}(x, y) = \vec{I}_0 + \tilde{a}_1(x, y) \cdot \vec{e}_1 + \dots + \tilde{a}_N(x, y) \cdot \vec{e}_N, \quad (11)$$

where, in general, for every z ($1 \leq z \leq N$), $\tilde{a}_z(x, y) \neq a_z(x, y)$. We want to estimate the Maximum Absolute Error corresponding to compression and decompression, i.e., the value

$$\max_{x, y, z} |I_z(x, y) - \tilde{I}_z(x, y)|.$$

Our objective is to provide a guaranteed upper bound for this MAE. Subtracting (11) from (10), we obtain

$$\begin{aligned} \vec{I}(x, y) - \vec{\tilde{I}}(x, y) &= (a_1(x, y) - \tilde{a}_1(x, y)) \cdot \vec{e}_1 + \dots \\ &+ (a_N(x, y) - \tilde{a}_N(x, y)) \cdot \vec{e}_N. \end{aligned} \quad (12)$$

Therefore, for each slice z , we have

$$\begin{aligned} I_z(x, y) - \tilde{I}_z(x, y) &= (a_1(x, y) - \tilde{a}_1(x, y)) \cdot e_{1,z} + \dots \\ &+ (a_N(x, y) - \tilde{a}_N(x, y)) \cdot e_{N,z}. \end{aligned} \quad (13)$$

As a result, for the absolute value of this difference, we can conclude that

$$|I_z(x, y) - \tilde{I}_z(x, y)| \leq$$

$$|a_1(x, y) - \tilde{a}_1(x, y)| \cdot |e_{1,z}| + \dots + |a_N(x, y) - \tilde{a}_N(x, y)| \cdot |e_{N,z}|. \quad (14)$$

For each slice z and for each horizontal location (x, y) , the difference

$$|a_z(x, y) - \tilde{a}_z(x, y)| \quad (15)$$

is bounded by the Maximum Absolute Error corresponding to slice z , i.e., by the value

$$MAE_z \stackrel{\text{def}}{=} \max_{x, y} |a_z(x, y) - \tilde{a}_z(x, y)|; \quad (16)$$

similarly, the absolute value $|e_{z,i}|$ of each component $e_{z,i}$ of the vector \vec{e}_z is bounded by the largest of these absolute values, i.e., in mathematical terms, by the l^∞ norm of this vector:

$$\|\vec{e}_z\|_\infty \stackrel{\text{def}}{=} \max(|e_{z,1}|, \dots, |e_{z,N}|). \quad (17)$$

If we use bounds (16) and (17), then, for any slice z and for any location (x, y) , we conclude that

$$\left| I_z(x, y) - \tilde{I}_z(x, y) \right| \leq MAE_1 \cdot \|\vec{e}_1\|_\infty + \dots + MAE_N \cdot \|\vec{e}_N\|_\infty. \quad (18)$$

Therefore, for its MAE, i.e., for the largest value of this absolute error for every (x, y) and z , we get a similar inequality:

$$MAE \leq MAE_1 \cdot \|\vec{e}_1\|_\infty + \dots + MAE_N \cdot \|\vec{e}_N\|_\infty. \quad (19)$$

The right-hand side of the inequality (19) is the desired guaranteed upper bound for the MAE of the compression distortion for the 3-D data set.

5. FORMULATION OF THE MINIMIZATION PROBLEM USING THE UPPER BOUND

In general, the guaranteed upper bound may exceed the actual value of MAE. Our numerical experiments show that, depending on the data, this upper bound can be up to 5 times larger than the actual value of the MAE. However, it is reasonable to expect that in general, if the upper bound can be made smaller, the actual MAE will also become smaller, and that larger upper bounds allow larger actual MAEs to be obtained.

To formulate the corresponding optimization problem, we will use, for each slice z , the mixed model for MAE_z . For convenience, let us denote the dependence of MAE_z on R_z by $f_z(R_z)$:

$$f_z(R_z) = MAE_z = \begin{cases} A_z \cdot \frac{1}{(R_z - R_{0z})^{\alpha_z}} & \text{if } R_z \leq \tilde{R}_z, \\ B_z \cdot 2^{-R_z} & \text{if } R_z > \tilde{R}_z. \end{cases} \quad (20)$$

In the direct optimization problem, we want to minimize overall MAE, and thus, (in view of the above empirical fact), we want to minimize the upper bound derived in the previous subsection. In other words, we want to minimize

$$\sum_{z=1}^N MAE_z \cdot \|\vec{e}_z\|_\infty = \sum_{z=1}^N f_z(R_z) \cdot \|\vec{e}_z\|_\infty, \quad (21)$$

under the condition that the average bit rate is R , i.e., that

$$\sum_{z=1}^N R_z = R \cdot N. \quad (22)$$

In the inverse optimization problem, we want to minimize the average bit rate (22) under the condition that the upper bound on the MAE does not exceed the given value MAE_0 . Clearly, the resulting MAE will be guaranteed to be below this target upper bound.

We use the Lagrange multiplier method to solve these problems. For the above constrained optimization problems, the Lagrange multiplier method leads to the following unconstrained problem:

$$Q \triangleq \sum_{z=1}^N f_z(R_z) \cdot \|\vec{e}_z\|_\infty + \lambda \cdot \sum_{z=1}^N R_z \rightarrow \min_{R_1, \dots, R_N}, \quad (23)$$

where λ is the Lagrange multiplier.

As shown in [5], also used in [4], minimization of Q can be accomplished through individual minimization of each entry $(f_z(R_z) \cdot \|\vec{e}_z\|_\infty + \lambda \cdot R_z)$ of the sum (23). Differentiating and setting to zero, the desired value R_z^o can be obtained as a solution of $f'_z(R_z^o) \cdot \|\vec{e}_z\|_\infty + \lambda = 0$. Thus, the optimal values R_z^o are such points where all the slopes of the functions $f'_z(R_z) \cdot \|\vec{e}_z\|_\infty$ are equal.

The dependence $f_z(R_z)$ is described by the formula (20). Differentiating this formula with respect to R_z , we conclude that:

$$f'_z(R_z) = \begin{cases} -A_z \cdot \alpha_z \cdot \frac{1}{(R_z - R_{0z})^{\alpha_z+1}} & \text{if } R_z \leq \tilde{R}_z \\ -B_z \cdot \ln 2 \cdot 2^{-R_z} & \text{if } R_z > \tilde{R}_z \end{cases} \quad (24)$$

If $R_z \leq \tilde{R}_z$, then the equation

$$f'_z(R_z) \cdot \|\vec{e}_z\|_\infty = -\lambda \quad (25)$$

leads to

$$R_z = R_{0z} + \left(\frac{A_z \cdot \alpha_z \cdot \|\vec{e}_z\|_\infty}{\lambda} \right)^{1/(\alpha_z+1)}. \quad (26)$$

If $R_z > \tilde{R}_z$, then the equation (25) leads to

$$R_z = \log_2(\ln 2 / \lambda) + \log_2(\|\vec{e}_z\|_\infty \cdot B_z). \quad (27)$$

For each z we want to choose one of the two solutions for R_z from equations (26) or (27) as the optimal choice for bit rate R_z . Our choice, however, will depend on the value of λ we are using to compute R_z . By combining two models together we are introducing the point of slope discontinuity (\tilde{R}_z), that means that we have a small interval of slope values (λ_L, λ_R) , $(\lambda_L > \lambda_R)$ that are not achievable.

If $\lambda \geq \lambda_L$ (this is true for the low bit rate part of the MM), then we return this value as the desired R_z^o . If $\lambda \leq \lambda_R$ (this is true for the the high bit rate part of the MM), then we return this value as the desired R_z^o . If λ is between the slopes λ_L and λ_R , then it is in the unachievable interval associated with the unique bit rate \tilde{R}_z and therefore we return $R_z^o = \tilde{R}_z$ as the desired bit rate.

Once we know the value of the Lagrange multiplier λ , we can determine the optimal values of the bit rates by using the above explicit formulas and applying the bisection method.

6. RESULTS AND CONCLUSIONS

The Battlescale Forecast Model [7] (BFM) data set available for use in this study consists of a cube of data for each of six physical variables (potential temperature T , pressure P , water vapor mixing ratio Wv and the U , V , and W components of the wind speed vector). For a specific variable Met , the cube $Met(z, x, y)$ is of dimensions $64 \times 129 \times 129$. The first dimension is the vertical height z , and the other two are x and y for the two horizontal spatial variables.

All the results are shown in percentages of the total amplitude range of the specific data cube. The results in the KLT domain are shown in Figure 3. As we can see, wherever the upper bound is smaller, the MAE is also smaller. Also, the resulting MAE is, on average, half as small as for the case when the same bit rate is used for all KLT slices.

Figure 4 shows the MAE results for all three strategies in more detail. The dashed line corresponds to the case where the same bit rate is used for all slices (OBR), the solid line corresponds to the solution obtained using the minimization of the upper bound on MAE. The parameter T is known to be the worst case in our experiments, however, even in this case we see some improvement in the achieved MAE. The dotted line corresponds to suboptimal solution obtained using the minimization of MSE. It clearly shows that even though this strategy allows us to achieve significant MSE reduction (see [3], [4]), this strategy is not the best for MAE.

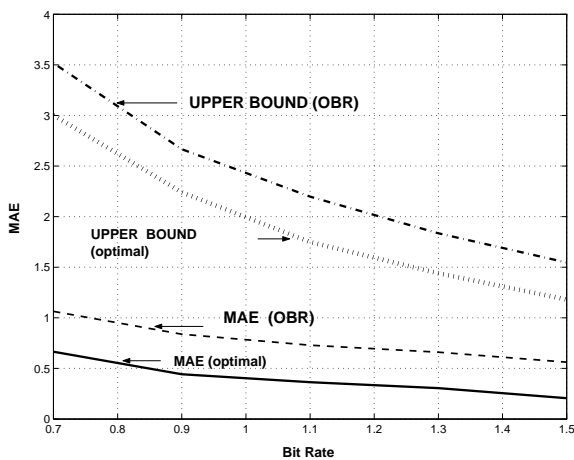


Fig. 3. T variable, using KLT pre-processing, MAE vs. upper bound (OBR vs. optimal solution)

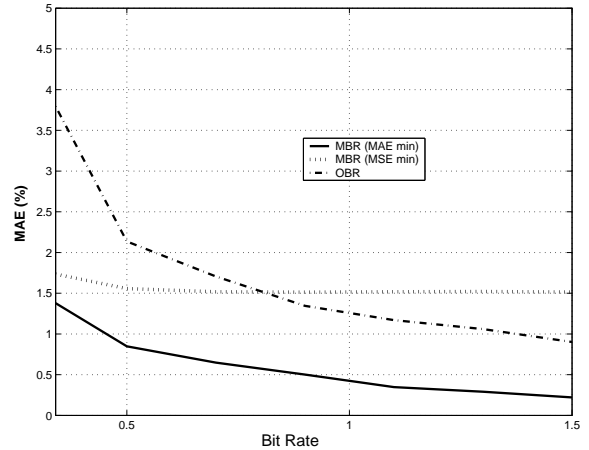


Fig. 4. T variable, using KLT pre-processing, MAE for suboptimal solution (MAE minimized, MSE minimized) vs. OBR.

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