

GEOMETRY OF COCKROACH PATHS: A POSSIBLE EXPLANATION

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Geometry of cockroach paths: an empirical fact and a need for an explanation. When attacked by a predator, how does a prey behave? A large number of studies of prey escape behavior was performed on easy-to-study prey species such as cockroaches.

A recent detailed analysis of the cockroach escape paths has shown that when escaping a predator, cockroaches prefer escape trajectories directed at 90, 120, 150, and 180 degrees from the direction of the attack (Domenici et al. 2008).

The authors of this study noticed that this is *not* the best strategy against a sophisticated (optimally behaving) predator: against such a predator, the best strategy is to follow a random escape trajectory. So, a natural question is: how to explain this behavior?

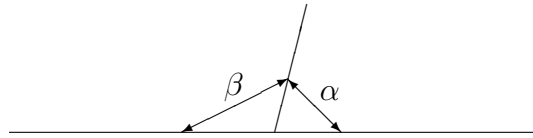
In this paper, we provide a simple geometric explanation for the observed cockroach behavior.

First approximation: a primitive predator. Let us start by considering the simplest possible (“primitive”) predator, a predator that moves in the direction towards the current position of the prey – without any sophisticated attempts to predict where the prey will be in the future moments of time.

A straightforward reaction to the primitive predator. A natural strategy to escape this predator is to run away as fast as possible along the same route, i.e., to escape in the direction which is 180 degrees from the direction to the predator.

A more sophisticated reaction to the primitive predator. A more sophisticated strategy is to choose an escape path which is as far away from the direction of attack as possible. A natural measure of a distance between the two rays (an escape path and a path of attack) is the angle. So, for each escape path, we have two possible angles

(distances) α and β depending on which direction the predator follows:



It is reasonable to select a path in which the worst-case distance $\min(\alpha, \beta)$ is as large as possible.

Since $\alpha + \beta = \pi$, both angles α and β cannot be $> \pi/2$, so one of them must be $\leq \pi/2$. Hence, we always have $\min(\alpha, \beta) \leq \pi/2$. If one of these angles is $< \pi/2$, then the minimum is $< \pi/2$; similarly, if one of the angles is $> \pi/2$, the other is $< \pi - \pi/2 = \pi/2$, and the minimum is $< \pi/2$. The only case when the minimum is $= \pi/2$ is when $\alpha = \beta = \pi/2$.

Thus, to attain the largest possible value of the worst-case distance from the predator trajectory, we must take $\alpha = \beta = \pi/2$ – i.e., follow the escape path which is at 90 degrees from the line of attack.

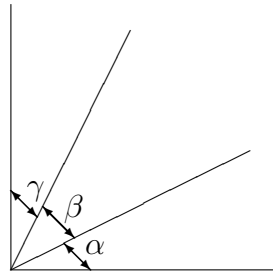


Comment. An alternative derivation of the need to follow either the straightforward escape route or the escape route which is orthogonal to the direction of the attack is given in the Appendix.

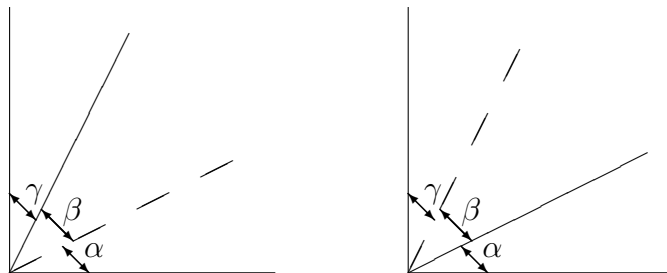
Second approximation: a more sophisticated predator. Let us now consider a more sophisticated (and thus, a more realistic) predators. These predators already know that a prey may run in the 180 and 90 degrees direction, so these directions may be already covered. Thus, the prey needs to select one or more additional directions.

Without loss of generality, let us concentrate on one of the four quadrants.

It is reasonable to select two more directions in this quadrant, so that if one of these directions is covered, some individuals can still escape. Let α be the angle to the first direction, β denote the angle between the selected directions, and γ denote the direction from the second selected direction to the orthogonal direction:



Optimal escape: a geometric formulation of the problem. For such predators, if a predator blocks both original escape routes and one of the two new escape routes, we have one of the two following situations:



Similarly to the case of the primitive predator, it is reasonable to maximize the worst-case distance of the escape trajectory from the predator-blocked routes. In the first situation, this distance is equal to $\min(\alpha, \beta)$; in the second situation, the distance is equal to $\min(\beta, \gamma)$. Thus, the worst-case distance is equal to

$$\min(\min(\alpha, \beta), \min(\beta, \gamma)) = \min(\alpha, \beta, \gamma).$$

Since $\alpha + \beta + \gamma = \pi/2$, all three angles α , β , and γ cannot be $> \pi/6$, so one of them must be $\leq \pi/6$. Hence, we always have $\min(\alpha, \beta, \gamma) \leq \pi/6$. If one of these angles is $< \pi/6$, then the minimum is $< \pi/6$. Similarly, if one of the angles is $> \pi/6$, the sum of the other two is $< \pi/2 - \pi/6 = \pi/3 = 2 \cdot (\pi/6)$, and thus, one of the remaining two angles is $< \pi/6$ – and the minimum is $< \pi/6$. The only case when the minimum is $= \pi/6$ is when $\alpha = \beta = \gamma = \pi/6 = 30^\circ$.

Thus, to attain the largest possible value of the worst-case distance from the predator-blocked trajectory, we must take $\alpha = \beta = \gamma = \pi/6$ – i.e., follow an escape path which is at 120 or at 150 degrees from the line of attack.

Summary: we have shown that a simple geometric model explains the observed escape paths of 90, 120, 150, and 180 degrees.

Open problems and future work. In this paper, we analyzed a reasonably simple type of predators and best ways to escape from them. The fact that these best ways turned out to be similar to the way actual species (cockroaches) evade predators makes us think that this approach has promise to explain other species' escape strategies as described, e.g., (Domenici 2002). In other words, it is desirable to provide a similar geometric analysis of other species' escape strategies.

References

P. Domenici, "Escape trajectory, ecological", In: A. H. El-Shaarawi and W. W. Piegorsch (eds.), *Encyclopedia of Environmental Metrics*, Vol. 2, John Wiley & Sons, Chichester, U.K., 2002, pp. 708–711.

P. Domenici, D. Booth, J. M. Blagburn, and J. P. Bacon, "Cockroaches Keep Predators Guessing by Using Preferred Escape Trajectories", *Current Biology*, 2008, Vol. 18, doi:10.1016/j.cub.2008.09.062

APPENDIX

What we plan to do. In this Appendix, we provide an alternative proof that when attacked by a primitive predator, it is reasonable to follow either the straightforward escape route or the escape route which is orthogonal to the direction of the attack. To provide this proof, let us provide an alternative reformulation of the escape problem in geometric terms.

Towards an alternative geometric formulation of the problem. It is natural to assume that, once the primitive predator starts going in the direction of the attack at the starting moment t_0 , it will continue moving in this direction for some time period, until some future moment of time t_f (at which its direction may change). In other words, a predator follows a straight line ℓ that connect its original location x_0 with the original location p of the prey. if we denote the predator's velocity by v , then we conclude that at the moment t_f , the predator will be at the point $x_0 + v \cdot (t_f - t_0)$.

Usually, the prey does not know the exact velocity v of the individual predator – this velocity depends on the agility, age, and other individual characteristics of the individual predator. However, we usually know the interval $[\underline{v}, \bar{v}]$ of possible values of this velocity. So, we can predict that at the moment t_f , the predator will be located somewhere within a segment $s = [\underline{s}, \bar{s}] = [x_0 + \underline{v} \cdot (t_f - t_0), x_0 + \bar{v} \cdot (t_f - t_0)]$.

A reasonable escape strategy for a prey is to move as far as possible from this dangerous segment s , i.e., to find the escape point e for which the distance

$$\rho(e, s) \stackrel{\text{def}}{=} \min\{\rho(e, x) : x \in s\}$$

is the largest possible.

The prey's ability to escape are limited by its own velocity. Let us denote the largest possible distance that a prey can travel during the time $t_f - t_0$ by d . Then, we arrive at the following geometric formulation of the escape problem:

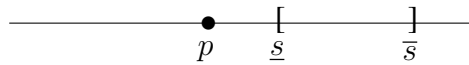
Resulting geometric formulation of the escape problem. On a plane, we have a straight line ℓ , a point $p \in \ell$, and a segment $[\underline{s}, \bar{s}] \subseteq \ell$. We are also given a real number $d > 0$. Among all the points within distance $\leq d$ from the given point p , we need to find the point e for which the distance $\rho(x, [\underline{s}, \bar{s}])$ is the largest possible.

Solution to the geometric problem. Let us show that the optimal escape point e :

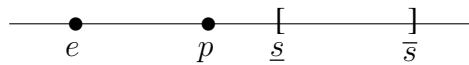
- is either located on the line ℓ – which corresponds to running away (direction 180 degrees),
- or has the property that the direction pe is orthogonal to ℓ – which corresponds to the 90 degrees direction.

It turns out that whether $e \in \ell$ or $ep \perp \ell$ depends on whether the point p is outside or inside the segment s .

First case: the original position p is outside the dangerous segment $s = [\underline{s}, \bar{s}]$. In this case, we have the following configuration:



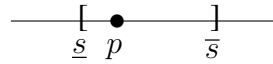
Due to the triangle inequality, we have $\rho(e, s) \leq \rho(e, p) + \rho(p, s)$, where the distance $\rho(p, s)$ from the original location d to the segment s is equal to $\rho(p, \underline{s})$, and $\rho(e, s) \leq d$. Thus, for every selection of the escape point e , we have $\rho(e, s) \leq d + \rho(p, \underline{s})$. The value $\rho(e, s) = d + \rho(p, \underline{s})$ is attained at the point on the line $p\underline{s}$ which is d units away from p :



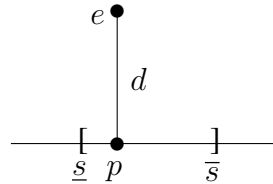
For all other points e , the distance $d(e, s)$ is smaller, since in the Euclidean space, for non-collinear points, the triangle inequality is strict.

So, in the first case, the best escape strategy is to run away in the 180 degrees direction.

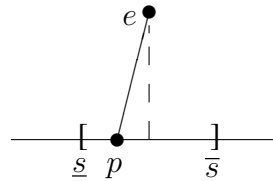
Second case: the original position p is inside the dangerous segment $s = [\underline{s}, \bar{s}]$. In this case, we have the following configuration:



In this case, we have $\rho(p, s) = 0$ hence, due to the triangle inequality, we have $\rho(e, s) \leq \rho(e, p) + \rho(p, s) \leq d + 0 = d$. The point at a distance d from p in the direction orthogonal to p is also at the distance d from the segment s :



For any other point, the direction pe will be “titled” and thus, the distance from e to s will be smaller than d :



So, in the second case, the best escape strategy is indeed to run away in the 90 degrees direction.

The statement is proven.