Why Are Students Risk-Prone

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Abstract— **Most education efforts are aimed at educating young people. So, to make education as effective as possible, it is desirable to take into account psychological features of young people. One of the typical features of their psychology – as distinct from the psychology of more mature population – is that they are much more risk-prone. To appropriately take this feature into account, we need first to explain it within a quantitative model. Such a basic explanation is provided in this paper.**

I. Formulation of the Problem

Young people are risk-prone. This is a lot of anecdotal evidence that young people are risk-prone. This was unexpectedly confirmed at one of our universities during a research project sponsored by the Texas Department of Transportation [1].

In traffic planning, it is important to take into account the driver's acceptance of risk. For example, in El Paso, there are two ways to get to the university from faraway places on the Westside:

- by the Interstate I-10 that passes through the city and
- by Mesa Street that also passes through this part of the city.

On average, I-10 is faster, but sometimes during rush hours, there are traffic jams or accidents there. If you are stuck on a freeway between two exists, it can take a while to get to the nearest exit.

On the other hand, on Mesa, even when there is an accident, there are usually side street that let you reach UTEP faster.

As a result, while on average, I-10 is faster, there is a low-probability risk that taking I-10 will drastically delay the travel time. So, whether a driver will take I-10 depends on his or her tolerance to risk.

Risk tolerance is known to be somewhat different at different geographical locations. So, to decide on the best traffic planning, the researchers decided to quantify to what extent the local population accepts risk.

At the university, we have more than 20,000 students – enthusiastic responders to different surveys, so the researchers decided to first ask the students. The results showed a drastic difference with risk-tolerance of drivers in different geographic locations: students showed full tolerance to risk. According to their responses, all they care about is the average travel time, and the possibility of long delays did not negatively affect their decision at all. Actually, the research got the result which is opposite to what one would expect based from a rational decision-maker: that the more risk, the more preferable the alternative. The researchers got the same result when they extended the survey to other young people, beyond the university students.

However, once the researchers repeated their survey with a general population, the results became fully in line with what was observed in other cities: that many people are risk-averse. The only non-risk-averse part of the population were young people.

They are so much non-risk-averse that they often seem to be risk-prone: they prefer a much risker route.

Our objective: use this feature in education. As university faculty, we are very much interested in effectiveness of the university education – and of course, when we prepare teachers for schools, we are also interested in making these future teachers as effective as possible.

The majority of students are young people. From this viewpoint, the more we know about the young people psychology, the better we can adjust our lesson so that they are most efficient. Being risk-prone is such a big part of the young people culture that it is definitely imperative to take this feature into account when developing teaching strategies.

Why are students risk-prone? To take this feature into account, it is desirable to understand it better. So, we arrive at a natural question: why are young people risk-prone?

To answer this question, let us analyze how people make decisions, and what is so different about young people.

II. Analysis of the Problem

Decision making: brief reminder. Let us recall what decision theory tells us re how decisions are made in the first place; see, e.g., [2], [3], [5], [7], [9]. Intuitively, a decision maker selects an alternative which is the best. So, to describe decision making in quantitative terms, we need to have a numerical description of how good different alternatives are to a given decision maker.

Decision theory is based on the following natural scale. We select two alternatives:

- a very bad alternative A_- and
- a very good alternative A_+,

so that every other alternative is in between A_- and A_+.

For every real number $p \in [0, 1]$, we can form a lottery $L(p)$ in which we get A_+ with probability p and A_- with the remaining probability $1 - p$. The larger the probability p of getting a good alternative, the more preferable the lottery. So, if $p < p'$, then $L(p) < L(p')$, where $A < A'$ means that

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to the decision maker, the alternative A' is better than the alternative A.

Let us now describe the quality of a given alternative A. When $p = 0$, the lottery $L(p)$ coincides with the very bad alternative A_- for which $A_- < A$, i.e., $L(0) < A$. When $p = 1$, the lottery $L(p)$ coincides with the very good alternative A_+ for which $A < A_+$, i.e., $A < L(1)$. Let $u(A)$ denote the supremum (least upper bound) of the set of all the values p for which $L(p) < A$. Then, one can show that:

- for all values $p < u(A)$, we have $L(p) < A$; and
- for all values $p > u(A)$, we have $A < L(p)$.

Indeed, let us assume that $p < u(A)$. In this case, the midpoint $\dfrac{p + u(A)}{2}$ is between p and $u(A)$:

$$p < \frac{p + u(A)}{2} < u(A).$$

Since $u(A)$ is the least upper bound of the set

$$\{q : L(q) < A\},$$

the smaller value $\dfrac{p + u(A)}{2}$ is not an upper bound for this set, i.e., there exists a value q for which $L(q) < A$ and $q \not\leq \dfrac{p + u(A)}{2}$, i.e., $\dfrac{p + u(A)}{2} < q$. From

$$p < \frac{p + u(A)}{2} < q,$$

we conclude that $p < q$ and thus, $L(p) < L(q)$. Hence $L(q) < A$ implies $L(p) < A$.

Similarly, when $p > u(A)$, the midpoint $\dfrac{p + u(A)}{2}$ is between $u(A)$ and p: $u(A) < \dfrac{p + u(A)}{2} < p$. Since $u(A)$ is the least upper bound for the set $\{q : L(q) < A\}$, and $\dfrac{p + u(A)}{2} \not\leq u(A)$, this means that $L\left(\dfrac{p + u(A)}{2}\right) \not< A$, i.e., $A \leq L\left(\dfrac{p + u(A)}{2}\right)$. From $\dfrac{p + u(A)}{2} < p$, we can now conclude that $L\left(\dfrac{p + u(A)}{2}\right) < L(p)$, so from $A \leq L\left(\dfrac{p + u(A)}{2}\right)$, we conclude that $A < L(p)$.

For each real number $\varepsilon > 0$, the alternative A is better than the lottery $L(u(A) - \varepsilon)$ and worse than the lottery $L(u(A) + \varepsilon)$. This is true for values which are as small as we want. It is thus reasonable to say that A is *equivalent* to $L(u(A))$; we will denote this equivalence by $A \sim L(u(A))$. The corresponding value $u(A)$ is called the *utility* of the alternative A.

Suppose now that we have a lottery L in which we get alternative A_1 with probability p_1, alternative A_2 with the probability p_2, …, and the alternative A_n with probability p_n. For every i from 1 to n, let $u(A_i)$ be the utility of the alternative A_i. What is the utility of the lottery L?

By definition of the utility, each alternative A_i is equivalent to the lottery $L(u(A_i))$ in which we get A_+ with probability $u(A_i)$ and A_- with the remaining probability $1 - u(A_i)$. Thus, the lottery L is equivalent to a two-stage lottery, in which:

- we first select i from 1 to n with probability p_i, and
- then, depending on the selection of i, select A_+ with probability $u(A_i)$ and A_- with the remaining probability.

As a result of this two-stage lottery, we get either A_+ or A_-. So, the utility of the lottery L is equal to the probability $u(L)$ of getting A_+ in this two-stage lottery. This probability can be estimated as the sum of the probabilities to get A_+ under the condition that different values $i = 1, \ldots, n$ were selected at the first stage:

$$u(L) =$$

$$p(1 \text{ selected at 1st stage} \,\&\, A_+ \text{ selected at 2nd stage}) +$$

$$\ldots +$$

$$p(n \text{ selected at 1st stage} \,\&\, A_+ \text{ selected at 2nd stage}).$$

Each of the corresponding probabilities can be described in terms of conditional probabilities:

$$p(i \text{ selected at 1st stage} \,\&\, A_+ \text{ selected at 2nd stage}) =$$

$$p(i \text{ selected at 1st stage}) \times$$

$$p(A_+ \text{ selected at 2nd stage} \,|\, i \text{ selected at 1st stage}),$$

i.e.,

$$p(i \text{ selected at 1st stage} \,\&\, A_+ \text{ selected at 2nd stage}) =$$

$$p_i \cdot u(A_i).$$

Thus, the overall probability $u(L)$ to get A_+ is equal to

$$u(L) = p_1 \cdot u(A_1) + \ldots + p_n \cdot u(A_n).$$

In mathematical terms, the right-hand side of this formula is the expected value of the utility $u(A_i)$. Thus, the utility of the lottery is equal to the expected value of the utilities of different alternatives.

Utility is not uniquely defined. The numerical value of the utility depends on the selection of two alternatives A_- and A_+. What happens if we select different alternatives? For example, what if we select alternatives $A'_- < A_-$ and $A'_+ > A_+$, and select lotteries $L'(p)$ and define utilities u' based on these new alternatives?

In this case, both original selections A_- and A_+ are equivalent to lotteries in terms of A'_- and A'_+, i.e., $A_- \sim L'(p_-)$ and $A_+ \sim L'(p_+)$ for some p_- and p_+.

By definition, a utility u of an event is the probability for which the event is equivalent to a lottery $L(p)$, a lottery in which A_+ appears with probability u and A_- appears with probability $1 - u$. Since:

- the original alternative A_- is, in its turn, equivalent to the lottery $L'(p_-)$ in which A'_+ appears with probability p_-, and
- the original alternative A_+ is, in its turn, equivalent to the lottery $L'(p_+)$ in which A'_+ appears with probability p_+,

the original event is equivalent to the lottery $L'(u')$, in which the new alternative A'_+ appears with probability

$$u' = u \cdot p_+ + (1-u) \cdot p_i.$$

Thus, when we change a scale, the new utility u' is a linear function of the old one:

$$u' = a \cdot u + b$$

for some $a > 0$ and b.

Strictly speaking, we only proved this for the case when $A'_- < A_-$ and $A'_+ > A_+$, but if we have two other scales, we can always compare each of them with a new scale in which $A''_- < A_-, A'_-$ and $A''_+ > A_+, A'_+$. In this case,

- transition from u to u'' is linear,
- transition from u'' to u is also linear, and thus,
- the transition from u to u' is linear as well – as a composition of two linear functions.

Vice versa, for every $a > 0$ and b, we can find the new alternatives A'_- and A'_+ for which $u' = a \cdot u + b$. Thus, the utility is defined modulo a linear transformation,

Comment. This non-uniqueness is similar to non-uniqueness in describing the numerical values of such quantities as time or temperature. Indeed, to describe different values of these quantities by numbers, we need to select:

- a starting point and
- a measuring unit.

Once we change the starting point and/or the measuring unit, we get different numerical values that are related to the original ones by a similar linear transformation $x' = a \cdot x + b$.

For example, the transformation from the temperature t_C in the Celsius scale and the temperature t_F in the Fahrenheit scale is described by the known formula $t_F = 1.8 \cdot t_C + 32$.

Decision making: summary. Each action has several possible consequences. Let n denote the number of such possible consequences, and let u_i denote the utility of the i-th consequence. Then, if p_i is the probability of the i-th consequence, we select an action for which the expected value $\sum\limits_{i=1}^{n} p_i \cdot u_i$ is the largest.

What we need to know to make a decision. According to the above description, to make a decision, for each possible consequence, we need to know two things:

- its utility u_i describing how beneficial this consequence is for the decision maker, and
- its probability p_i.

The utility u_i describes to what extent the outcome is beneficial for the decision maker, something that any decision maker, experienced or not, can judge for him- or herself. On the other hand, the probability of different events is something about which we can have more knowledge or less knowledge, and how well we know these probabilities depends on our experience.

How can we determine the probabilities of different consequences? A usual way to find the probability p_i of an event is to make several (N) observations and to estimate p_i as the frequency with which this event occurs, i.e., as the ratio $p_i \approx \widetilde{p}_i \stackrel{\text{def}}{=} \dfrac{N_i}{N}$, where N_i is the number of cases when the i-th event occurred.

It is known (see, e.g., [10]) that the expected value of this estimate is $p_i = E[\widetilde{p}_i]$, and the standard deviation is equal to

$$\sigma_i = \sqrt{E[(\Delta p_i)^2]} = \sqrt{\frac{p_i \cdot (1-p_i)}{N}},$$

where we denoted $\Delta p_i \stackrel{\text{def}}{=} \widetilde{p}_i - p_i$. The values Δp_i corresponding to different events are, for large n, practically independent.

How the uncertainty of these estimates affects the decision making. Let us describe how the uncertainty of these events affect decision-making. In the ideal world, we should take into account the actual probabilities p_i, and base our decisions based on the expected utility $u = \sum\limits_{i=1}^{n} p_i \cdot u_i$.

In reality, we only know the approximate values $\widetilde{p}_i$ of the probabilities, i.e., we know that $p_i = \widetilde{p}_i - \Delta p_i$ for some random (unknown) differences Δp_i. Substituting this expression into the above formula for utility, we conclude that

$$u = \sum_{i=1}^{n} p_i \cdot u_i = \sum_{i=1}^{n} \widetilde{p}_i \cdot u_i - \sum_{i=1}^{n} \Delta p_i \cdot u_i,$$

i.e.,

$$u = \widetilde{u} - \Delta u,$$

where

$$\widetilde{u} \stackrel{\text{def}}{=} \sum_{i=1}^{n} \widetilde{p}_i \cdot u_i$$

and

$$\Delta u \stackrel{\text{def}}{=} \sum_{i=1}^{n} \Delta p_i \cdot u_i.$$

For many practical problems, the number of alternatives n is large. For such problems, the value Δu is a linear combination of a large number of independent random variables Δp_i. According to the Central Limit Theorem (see, e.g., [10]) this implies that the distribution of Δu is close to normal. Since the mean value of each Δp_i is 0, the mean value of the linear combination is also 0, and its variance is equal to

$$E[(\Delta u)^2] = \sum_{i=1}^{n} E[(\Delta p_i)^2] \cdot u_i^2,$$

i.e., using the known formulas for $E[(\Delta p_i)^2]$,

$$E[(\Delta u)^2] = \sum_{i=1}^{n} \frac{p_i \cdot (1-p_i)}{N} \cdot u_i^2.$$

For most of the events, the probabilities p_i are small, so in the first approximation, $p_i \cdot (1-p_i) \approx p_i$, and

$$E[(\Delta u)^2] = \frac{1}{N} \cdot \sum_{i=1}^n p_i \cdot u_i^2.$$

So, the standard deviation $\sigma = \sqrt{E[(\Delta u)^2]}$ is equal to

$$\sigma \approx \frac{1}{\sqrt{N}} \cdot \sqrt{\sum_{i=1}^n p_i \cdot u_i^2}.$$

Thus, the only information that we have about the actual (unknown) value of the expected utility u of an action is that u is (approximately) normally distributed with a known mean $\widetilde{u}$ and a known standard deviation σ.

It is known that for a normal distribution with mean a and standard deviation σ:

- with probability 90%, the actual value of the random variable is in the interval $[a-2\sigma, a+2\sigma]$;
- with probability 99.9%, the actual value of the random variable is in the interval $[a-3\sigma, a+3\sigma]$;
- with probability $1-10^{-8}$, the actual value of the random variable is in the interval $[a-6\sigma, a+6\sigma]$;
- etc.

Thus, based on N observations, we can conclude, with certain confidence, that the actual (unknown) value of the expected utility u belongs to the interval $[\widetilde{u}-k_0\cdot\sigma, \widetilde{u}+k_0\cdot\sigma]$, where

- for $k_0 = 2$, we get confidence 90%;
- for $k_0 = 3$, we get confidence 99.9%;
- for $k_0 = 6$, we get confidence $1-10^{-8}$, etc.

So, instead of the *exact* values of the utility, we now have an *interval* of possible values of the utility. How can we make decisions based on such intervals?

Decision making under interval uncertainty: brief reminder. In the previous section, we encountered a situation in which we do not know the exact value u of the utility, we only know that this value belongs to the interval $[\underline{u}, \overline{u}]$. The problem of decision making under such interval uncertainty was first handled by the future Nobelist L. Hurwicz in [4].

As we have mentioned earlier, the preference of each situation can be described by a utility value. Thus, to describe decisions under interval uncertainty, we must assign, to each such interval $[\underline{u}, \overline{u}]$, a utility value $u(\underline{u}, \overline{u})$.

No matter what value we get from this interval, this value will be larger than or equal to $\underline{u}$ and smaller than or equal to $\overline{u}$. Thus, the equivalent utility value $u(\underline{u}, \overline{u})$ must satisfy the same inequalities:

$$\underline{u} \le u(\underline{u}, \overline{u}) \le \overline{u}.$$

In particular, for $\underline{u} = 0$ and $\overline{u} = 1$, we get

$$0 \le \alpha \le 1,$$

where we denoted $\alpha \stackrel{\rm def}{=} u(0,1)$.

We have mentioned that the utility is determined modulo a linear transformation $u' = a\cdot u + b$. It is therefore reasonable to require that the equivalent utility does not depend on what scale we use, i.e., that for every $a > 0$ and b, we have

$$u(a\cdot \underline{a}+b, a\cdot\overline{u}+b) = a\cdot u(\underline{u}, \overline{u}) + b.$$

In particular, for $\underline{u} = 0$ and $\overline{u} = 1$, we get

$$u(b, a+b) = a\cdot u(0,1) + b = a\cdot\alpha + b.$$

So, for every $\underline{u}$ and $\overline{u}$, we can take $b = \underline{u}$, $a = \overline{u} - \underline{u}$, and get

$$u(\underline{u}, \overline{u}) = \underline{u} + \alpha\cdot(\overline{u}-\underline{u}) = \alpha\cdot\overline{u} + (1-\alpha)\cdot\underline{u}.$$

This expression is called *Hurwicz optimism-pessimism criterion*, because:

- when $\alpha = 1$, we make a decision based on the most optimistic possible values $u = \overline{u}$;
- when $\alpha = 0$, we make a decision based on the most pessimistic possible values $u = \underline{u}$;
- for intermediate values $\alpha \in (0,1)$, we take a weighted average of the optimistic and pessimistic values.

It is worth mentioning that most people are more optimists than pessimists in the sense that the weight α of the optimistic case is usually larger than the weight $1-\alpha$ of the pessimistic case: $\alpha < 1-\alpha$, i.e., equivalently, $2\cdot\alpha > 1$ and $\alpha > 0.5$.

Let us apply Hurwicz criterion to our problem. In our case, $\underline{u} = \widetilde{u} - k_0\cdot\sigma$ and $\underline{u} = \widetilde{u} + k_0\cdot\sigma$, where

$$\widetilde{u} = \sum_{i=1}^n \widetilde{p}_i \cdot u_i.$$

Hence, the equivalent utility is equal to

$$\alpha\cdot\overline{u} + (1-\alpha)\cdot\underline{u} = \alpha\cdot(\widetilde{u}-k_0\cdot\sigma) + (1-\alpha)\cdot(\widetilde{u}+k_0\cdot\sigma) =$$

$$\widetilde{u} + (2\alpha-1)\cdot k_0\cdot\sigma,$$

where $k_0 = 2$, 3, or 6 (or some other similar number, depending on the desired confidence level), and

$$\sigma \approx \frac{1}{\sqrt{N}} \cdot \sqrt{\sum_{i=1}^n p_i \cdot u_i^2}.$$

This formula enables us to produce the desired explanation of why young people are risk-prone.

III. Resulting Explanation and How to Use It

In these terms, what distinguishes young people from others. The main difference between young people and the general population is that the young people are less experienced, i.e., in our terms, they have encountered few situations N.

Consequences of the above formula for the general population. For people with experience, the value N is large, thus σ is small, and the resulting effective utility $u \approx \widetilde{u}$ is determined by the usual expected utility formula $\sum\limits_{i=1}^n \widetilde{p}_i \cdot u_i$.

Consequences for young people. For young people, the value N is small, so we can no longer ignore the σ terms in comparison with the expected utility term $\widetilde{u}$.

In the extreme case, when the σ-term is dominant, we select the alternative for which this term is the largest, i.e., equivalently, for which the expected value $\sum_{i=1}^{n} p_i \cdot u_i^2$ of the *squared* utility is the largest.

And here is where risk-proneness comes into picture. Let us assume, e.g., that we are talking about monetary outcomes, and that the utility value is proportional to the monetary amount. Let us assume that we have 10 different alternatives the probability of each of which is 0.1. Let us consider the following two situations.

In the first situation, in each alternative, the person get the amount 0.5. In this situation, there is no risk, we get 0.5 no mater what. In this case, the expected utility is 0.5, and

$$\sum_{i=1}^{n} p_i \cdot u_i^2 = 0.25.$$

In the second situation, in the first five alternatives, the person gets the amount 0, and in the second five alternatives, she gets the utility 1. In this situation, there is a high risk: with probability 0.5, we get nothing. The expected utility is still the same

$$5 \cdot 0.1 \cdot 0 + 5 \cdot 0.1 \cdot 1 = 0.5,$$

but now we have

$$\sum_{i=1}^{n} p_i \cdot u_i^2 = 5 \cdot 0.1 \cdot 0 + 5 \cdot 0.1 \cdot 1 = 0.5.$$

From the viewpoint of the sum $\sum_{i=1}^{n} p_i \cdot u_i^2$, the second (risk-prone situation) is clearly preferable to the previous one, so a young person will prefer it. Similarly, if we take into account both terms, the risk-prone strategy is clearly preferable. This explains why young people are risk-prone.

In more general mathematical terms, when we maximize the expected value of some function $y = f(x)$ of the monetary value x, risk-averse means that a person prefers to receive the average $\sum_{i=1}^{n} p_i \cdot x_i$ with probability 1 rather than participate in the lottery in which she gets x_i with probability p_i:

$$f\left(\sum_{i=1}^{n} p_i \cdot x_i\right) \geq \sum_{i=1}^{n} p_i \cdot f(x_i).$$

Functions with this property are called *concave*. In contrast, for the function $f(x) = x^2$, the opposite inequality is true because this function is *convex*. The fact that we naturally got a convex function shows that young people are indeed risk-prone.

How we can use this conclusion: ideas. Based on the fact that young people prefer risky situations, in which the results depend on a random selection, a good strategy is to introduce as much randomness into teaching as possible, e.g.:

- use surprise quizzes – in addition to normally scheduled tests and quizzes;
- for class activities, group students into randomly selected groups – instead of trying to group them into most effective groups;
- assign larger homeworks, with more problems than a Teaching Assistant and/or a professor can grade before the next class – with an understanding that only problems with randomly selected numbers will be graded.

Possibilities are unlimited, and, as our experience shows, excitement (and hence improvement) is guaranteed.

Future work and need to add fuzzy knolwedge. Our main focus was on the explanation. Of course, since we have a quantitative model, the next step would not just to give qualitative recommendations, but to use his model to provide *quantitative* recommendations: how much randomness should be introduce to make education most efficient.

It is also desirable to take into account that our knowledge about the probabilities p_i of different alternatives comes not only from observations, it also comes from the expertise of other people. This expertise is often formulated not in precise numerical terms, but rather by words from natural language (like "very rare"). Fuzzy logic [6], [8], [11] is a natural successful way of formalizing such knowledge. Thus, it is desirable to add such fuzzy knowledge to the above statistical model.

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