How to Modify Grade Point Average (GPA) to Make It More Adequate

Joe Lorkowski[1], Olga Kosheleva[2], and Vladik Kreinovich[1]
[1]Department of Computer Science
[2]Department of Teacher Education
University of Texas at El Paso
500 W. University
El Paso, TX 79968, USA
lorkowski@computer.org, olgak@utep.edu, vladik@utep.edu

Abstract

At present, the amounts of knowledge acquired by different graduates of the same program are usually compared by comparing their GPAs. We argue that this is not always the most adequate description: for example, if, after completing all required classes with the highest grade of "excellent" (A), a student takes an additional challenging class and gets a "satisfactory" grade (C), the amount of her knowledge increases, but the GPA goes down. We propose a modification of the GPA which is free of this drawback and is, thus, more adequate for describing the student's knowledge. We also provide a psychological explanation for why people cling to the traditional GPA.

How graduate's knowledge is compared now. At present, the amounts of knowledge acquired by different graduates of the same program are usually compared by comparing their Grade Point Average (GPA). A GPA is simply an arithmetic average of all the grades that a student got in different classes – usually weighted by the number of credit hours corresponding to each class.

Usually, two types of GPA are considered:

- the overall GPA that counts all the classes that the student took at the university, and
- the major GPA, in which general education classes are not counted, the only classes which are counted as classes directly related to the student's major.

Why this is not always adequate. Let us give an example explaining why this may not be the most adequate way of comparing the students' knowledge.

Let us assume that we have two students; both took all the classes required for a Computer Science degree, and both got "excellent" grades (As) in all these classes.

The first student did not take any other classes, while the second student decided to also take an additional – very challenging – class, and got a satisfactory grade of C in this additional class.

From the common sense viewpoint, the second student knows everything that the first student knows, plus she also knows some additional material that she learned in this challenging class. However, the GPA is higher for the first student, so, from the usual GPA viewpoint, the first student is academically better.

How can we modify the GPA to make it more adequate: a proposal. To avoid the above counterintuitive situation, we propose the following natural modification of the GPA:

- if the student took only the classes needed for graduation, then his/her GPA is computed in exactly the same way as usual;
- if a student took an additional class which is not related to his/her major and is not required for his/her degree, the grade for this class should be simply ignored;
- if for some topic needed for the major, the student took more classes than required, then only the the required number of top grades are counted when computing the modified GPA.

For example, if, instead of required three technical electives, a student took four classes and got grades A, C, A, and B, then only the top three grades (two As and a B) are counted.

Why this works. If a student, in addition to all As for required classes, gets a C for an addition non-required technical elective, this C grade will not count towards a newly modified GPA.

If this is a good idea, why is it not used? In his well-known book on the actual human reasoning [1], a Nobelist Daniel Kahneman cites an example of a seemingly irrational human behavior.

Namely, according to Chapter 15, Section 15.1 "Less Is More, Sometimes Even in Joint Evaluation", when pricing two large dinnerware sets,

- one consisting of 24 pieces in perfect condition, and
- the other consisting of the same 24 pieces plus 16 broken ones,

most people value the second set lower.

Rationally, this makes no sense, since after buying the second set, we can simply throw away the broken pieces.

According to Kahneman, this is an example of a general feature of human reasoning. According to [1], when in a hurry, people often use an arithmetic average as a substitute for the sum; see, e.g., Chapter 8, Section 8.2 "Sets and Prototypes".

For example, when people are asked to compare two pictures with multiple line segments in each of them, and decide in which of the two pictures, the *total* length of its line segments is larger, they usually select the picture with the largest *average* length.

This probably explains the dinnerware sets comparison – that people compare *average* values of pieces in two sets instead of comparing the overall values of these two sets: for the second set, the average value is indeed lower.

This probably also explains why the traditional GPA – an arithmetic average – is used instead of more adequate modifications – like the one we are proposing in this paper.

But why do people estimate an average: our possible explanation. Our explanation for the use of arithmetic averages is that the arithmetic average is much easier to compute than, e.g., the sum.

This may sound somewhat counter-intuitive, because, at first glance, the formula for the arithmetic average $\overline{x} = \dfrac{x_1 + \ldots + x_n}{n}$ looks somewhat more complex to compute than the formulas for the sum $s = x_1 + \ldots + x_n$: to compute the average, we need to perform all the additions needed for the sum plus one additional division.

This is indeed the case if we talk about *exact* computations: to compute the exact sum or the exact average, one needs to process each of n numbers at least once – if we do not process one of the numbers, we cannot get the exact value of sum or average. Since each elementary arithmetic operation takes at most two numerical inputs, this means that in both cases, we need at least $n/2 = O(n)$ operations.

However, if we take into account that the values x_i are only known approximately and that, as a result, we only need approximate values of sum and average, then the computational complexity changes. For the sum, we still need to count the intervals, but to compute the approximate values of the average, we can use Monte-Carlo techniques: namely, we can select a random sample of values and take the arithmetic average of this sample.

According to the Large Numbers Theorem, when the sample size is large, this random-sample-based arithmetic average provides a good approximation to the desired exact average – and the larger the sample, the more accurate this approximation; see, e.g., [2].

The required sample size – and thus, the corresponding computational complexity of estimating the average this way – depends only on the desired accuracy of estimating the average, and does not depend on the number n of original values. Thus, for a fixed accuracy, the computational complexity of this algorithm does not grow with n at all, it is $O(1)$, while the complexity of computing the sum still grows with n as $O(n)$. Since for large n, $O(1) \ll O(n)$, this explains why people use an average as a substitute for the sum.

1 Acknowledgment

This work was supported in part by the National Science Foundation grants HRD-0734825 and HRD-1242122 (Cyber-ShARE Center of Excellence) and DUE-0926721.

References

[1] D. Kahneman, *Thinking, Fast and Slow*, Farrar, Straus, and Giroux, New York, 2011.

[2] D. J. Sheskin, *Handbook of Parametric and Nonparametric Statistical Procedures*, Chapman & Hall/CRC, Boca Raton, Florida, 2011.