Positive Consequences of Negative Attitude: Game-Theoretic Analysis

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Abstract

At first glance, the world would be a better place if all the people had positive attitude towards each other. It is known that this is not always the case: sometimes, the excess of positive attitude can lead to negative consequences. In this paper, we show that, vice versa, a reasonable amount of negative attitude can make life better for everyone. What is therefore needed is not the exclusive appearance of positive attitude, but rather a balance – titled towards moderately positive attitude.

1 How to Take Attitude into Account When Describing Human Behavior

How to describe human behavior. According to decision theory, a rational (i.e., consistent) human decision maker should select an alternative with the largest possible value of expected utility u; see, e.g., [5, 9, 10, 12, 16].

How to take attitude into account. Usually, it is assumed that the utility of a person depends only on this person's gains and losses. However, in real life, the person's preferences also take into account gains and losses of others. If a person A has a positive attitude towards a person B, then B's happiness (and high utility values) increases A's utility as well. On the other hand, if a person A has a negative attitude towards a person C, then C's happiness makes the person A less happy (i.e., decreases A's utility value).

The idea that a utility of a person depends on utilities of others was first described in [14, 15]. It was further developed by a future Nobelist Gary Becker; see, e.g., [1]; see also [3, 6, 7, 17].

In general, the utility u_i of i-th person can be described as $u_i = f_i(u_i^{(0)}, u_j)$, where $u_i^{(0)}$ is the utility that does not take other people into account, and u_j are utilities of other people.

This dependence is usually small, so we expand it into Taylor series and keep only the linear terms of this expansion. In this case, the utility u_i of i-th person is equal to

$$u_i = u_i^{(0)} + \sum_{j \neq i} \alpha_{ij} \cdot u_j,$$

where each coefficient a_{ij} describes the i-th person's attitude towards the j-th one.

Excessive positive attitude can make people unhappy: a known fact. At first glance, life would be perfect if everyone has very positive attitude towards each other. Alas, a simple analysis shows that excessive positiveness can be harmful [2, 4, 8, 12].

Let us start with the case of mutual positive attitude between persons P_1 and P_2, meaning that $\alpha_{12} > 0$ and $\alpha_{21} > 0$. In this case, we have

$$u_1 = u_1^{(0)} + \alpha_{12} \cdot u_2;$$

$$u_2 = u_2^{(0)} + \alpha_{21} \cdot u_1.$$

Once we know the original utility values $u_1^{(0)}$ and $u_2^{(0)}$, we can solve this system of linear equations and find the resulting values of utility:

$$u_1 = \frac{u_1^{(0)} + \alpha_{12} \cdot u_2^{(0)}}{1 - \alpha_{12} \cdot \alpha_{21}};$$

$$u_2 = \frac{u_2^{(0)} + \alpha_{21} \cdot u_1^{(0)}}{1 - \alpha_{12} \cdot \alpha_{21}}.$$

As a result, when we have excessive positive attitude, e.g., when we have $\alpha_{12} > 1$ and $\alpha_{21} > 1$, then positive original pleasures $u_i^{(0)} > 0$ lead to $u_i < 0$ – i.e., to unhappiness.

According to [2, 4, 8, 12], this phenomenon may be one of the reasons why people in love often experience deep negative emotions.

If we consider $n > 2$ persons, then even less excessive positive attitude can lead to negative emotions. Let us illustrate this on a simple case when we everyone has the same positive attitude $\alpha > 0$ towards each other, and when the original utilities are the same: $u_i^{(0)} = u^{(0)} > 0$. In this case, the actual utility u of each person is determined by the equation

$$u_i = u_i^{(0)} + \alpha \cdot \sum_{j \neq i} u_j,$$

i.e., in this case,

$$u = u^{(0)} + \alpha \cdot (n - 1) \cdot u.$$

Thus, here,

$$u = \frac{u^{(0)}}{1 - \alpha \cdot (n - 1)},$$

and is, thus, negative when $\alpha > \dfrac{1}{n-1}$.

What we do in this paper. The above examples show that excessive (or even not so excessive) positive attitude can make people unhappy. In this paper, we show that, vice versa, negative attitude can make people happier.

This may be the evolutionary reason for negative attitude?

2 Positive Consequences of Negative Attitude: Two Examples

First example: negative attitude helps to decrease the negative effect of excessive positive attitude. Let us consider a simple case of a person P_0 in the presence of two "saints" P_1 and P_2 with excessive positive attitude towards everyone: $\alpha_{12} = \alpha_{10} = \alpha_{21} = \alpha_{20} = \alpha > 1$. Let us also assume that the original utilities are the same for all three persons: $u_0^{(0)} = u_1^{(0)} = u_2^{(0)} = u_0^{(0)}$.

We already know that if P_0 will also have excessive positive attitude towards everyone, all three will turn out to be unhappy. What attitude β should we recommend to P_0 towards P_1 and P_2 so that at least this person will be happy?

Of course, we can recommend $\beta = 0$, this will guarantee that $u_0 = u_0^{(0)} > 0$, but can we do better? Let us analyze the problem. Here, since we assume P_1 and P_2 to be similar, their utilities are equal, so $u_1 = u_2$. Thus, we have

$$u_0 = u^{(0)} + 2\beta \cdot u_1;$$

$$u_1 = u^{(0)} + \alpha \cdot u_0 + \alpha \cdot u_1.$$

From the second equation, we conclude that $u_1 \cdot (1 - \alpha) = u^{(0)} + \alpha \cdot u_0$, so

$$u_1 = \frac{1}{1-\alpha} \cdot u^{(0)} + \frac{\alpha}{1-\alpha} \cdot u_0.$$

Substituting this expression into the first equation, we conclude that

$$u_0 = u^{(0)} + \frac{2\beta}{1-\alpha} \cdot u^{(0)} + \frac{2\alpha \cdot \beta}{1-\alpha} \cdot u_0.$$

By moving all the terms containing u_0 into one side and all the terms containing $u^{(0)}$ into the other side, we get

$$u_0 \cdot \left(1 - \frac{2\alpha \cdot \beta}{1-\alpha}\right) = u^{(0)} \cdot \left(1 + \frac{2\beta}{1-\alpha}\right),$$

hence

$$u_0 = \frac{1 + \dfrac{2\beta}{1-\alpha}}{1 - \dfrac{2\alpha \cdot \beta}{1-\alpha}} \cdot u^{(0)}.$$

To check how changing β from $\beta = 0$ will affect this utility value, let us compute the derivative of this value with respect to β when $\beta = 0$. This derivative is equal to

$$\frac{du_0}{d\beta} = \frac{2\beta}{1-\alpha} + \frac{2\alpha\cdot\beta}{1-\alpha} = \frac{2\cdot(1+\alpha)\cdot\beta}{1-\alpha}.$$

When $\alpha > 1$, this derivative is negative, so decreasing β below 0 – i.e., adapting a negative attitude – indeed makes P_0 happier.

So, in this example, a negative attitude makes P_0 happier.

Second example: negative attitude enhances fairness. Another example refers to joint decision making. In 1950, another future Nobelist John Nash showed that under reasonable assumptions like symmetry, independence from irrelevant alternatives, and *scale invariance* (i.e., invariance under replacing the original utility function $u_i(A)$ with an equivalent function $a\cdot u_i(A)$), the only appropriate group decision rule is selecting an alternative A for which the product $\prod\limits_{i=1}^{n} u_i(A)$ is the largest possible [11]; see also [9, 10, 12, 13].

Here, the utility functions must be scaled in such a way that the "status quo" situation $A^{(0)}$ is assigned the utility 0. This re-scaling can be achieved, e.g., by replacing the original utility values $u_i(A)$ with re-scaled values $u'_i(A) \stackrel{\text{def}}{=} u_i(A) - u_i(A^{(0)})$.

When we maximize the product, there is no guarantee that the solution will be in any sense fair, i.e., that in some sense, the resulting utilities will be close to each other. However, if we consider the case when the two participants have some negative feelings towards each other, then we get some fairness. Indeed, in this case, the utility of the first person is $u_1 = u_1^{(0)} - |\alpha_{12}|\cdot u_2$, and the utility of the second person is $u_2 = u_2^{(0)} - |\alpha_{21}|\cdot u_1$, for some $|\alpha_{ij}| \in (0,1)$.

In this case, the above formulas imply that

$$u_1 = \frac{u_1^{(0)} - |\alpha_{12}|\cdot u_2^{(0)}}{1-\alpha_{12}\cdot\alpha_{21}}$$

and

$$u_2 = \frac{u_2^{(0)} - |\alpha_{21}|\cdot u_1^{(0)}}{1-\alpha_{12}\cdot\alpha_{21}}.$$

A joint decision requires that both sides agree, and for both to agree, each participant's utility should be larger than (or at least equal to) the status-quo utility 0. Thus, in any joint decision, we will have $u_1 \ge 0$ and $u_2 \ge 0$, hence $u_1^{(0)} \ge |\alpha_{12}|\cdot u_2^{(0)}$ and $u_2^{(0)} \ge |\alpha_{21}|\cdot u_1^{(0)}$. So, the ratio between the gains $u_i^{(0)}$ cannot be very much different from 1:

$$|\alpha_{21}| \le \frac{u_2^{(0)}}{u_1^{(0)}} \le \frac{1}{|\alpha_{12}|}.$$

The more negative is the mutual attitude, the more restrictive this condition – and in the limit when $|\alpha_{12}|$ and $|\alpha_{21}|$ tend to 1, we get perfect equality $u_1^{(0)} = u_2^{(0)}$.

So, in this example too, negative attitude leads to positive consequences.

Acknowledgments

This work was supported in part by the US National Science Foundation grants HRD-0734825, HRD-1242122, and DUE-0926721.

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