

Science Is Helpful for Engineering Applications: A Theoretical Explanation of an Empirical Observation

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Abstract

Empirical evidence shows that when engineering design uses scientific analysis, we usually get a much better performance than for the system designed by using a trial-and-error engineering approach. In this paper, we provide a quantitative explanation for this empirical observation.

1 An Empirical Observation

Observation. At a recent annual conference of the American Society of Mechanical Engineers, one of the plenary talks was given by Dr. Hai Wang from Stanford University [4]. In his talk, Dr. Wang emphasized that scientific research is important for engineering practice. His argument was that without such a research, engineers tend to concentrate on one or few parameters of the system, and experience shows that much better performance can be achieved in a scientific approach, when all the parameters of the science can be changed.

A similar idea was emphasized by Dr. Delbert Tesar from the University of Texas at Austin in his plenary talk at the same conference [3]. Specifically, Dr. Tesar emphasized that engineers tend to follow one – most “fashionable” – direction, i.e., modify the values of only one (or few) parameters of the systems, and this severely limits their ability to reach the most efficient solutions to practical problems.

What we do in this paper. The purpose of this paper is to come up with a quantitative explanation of why the traditional trial-and-error engineering approach indeed leads to a drastic decrease in efficiency – as compared to all-variables optimization corresponding to the scientific approach.

2 Our Explanation

Mathematical model of the situation: a general description. Let $x_1, \dots, x_n, \dots, x_N$ be parameters of the design, and let $f(x_1, \dots, x_n, \dots)$ be the value of the efficiency corresponding to the design parameters x_1, \dots, x_n, \dots .

Without losing generality, we can assume that the parameters are sorted by the degree of their influence on efficiency: the parameter x_1 is the most influential, the parameter x_2 is the second influential, etc.

The influence of each parameter x_i on the efficiency $f(x_1, \dots, x_n, \dots)$ can be measured by the corresponding partial derivative $c_i \stackrel{\text{def}}{=} \frac{\partial f}{\partial x_i}$. In these terms, the above assumption takes the form

$$c_1 \geq c_2 \geq \dots \geq c_n \geq \dots$$

Let us now describe how the scientific approach and the trial-and-error engineering approach compare in this model.

Scientific approach: a formal description. In the scientific approach, we can modify all the parameters to maximize the efficiency. A natural maximization step involving all the parameters is the gradient ascent step, in which we replace each original value x_i with a modified value $x_i + \lambda \cdot \frac{\partial f}{\partial x_i} = x_i + \lambda \cdot c_i$, for some appropriate step λ .

After this step, in a linear approximation, the value of the efficiency changes to $f(x_1, \dots, x_n)$ to

$$\begin{aligned} f(x_1 + \lambda \cdot c_1, \dots, x_n + \lambda \cdot c_n, \dots) &= f(x_1, \dots, x_n, \dots) + \sum_{i=1}^N \frac{\partial f}{\partial x_i} \cdot (\lambda \cdot c_i) = \\ &= f(x_1, \dots, x_n, \dots) + \lambda \cdot \sum_{i=1}^N c_i^2. \end{aligned}$$

So, the corresponding increase in efficiency

$$\Delta_s \stackrel{\text{def}}{=} f(x_1 + \lambda \cdot c_1, \dots, x_n + \lambda \cdot c_n, \dots) - f(x_1, \dots, x_n, \dots)$$

is equal to

$$\Delta_s = \lambda \cdot \sum_{i=1}^N c_i^2. \quad (1)$$

A trial-and-error engineering approach: a formal description. In the trial-and-error engineering approach, we only modify one (or a few) most promising parameters. Let k denote the number of the modified parameters; then $k \ll N$.

In this case, we only experiment with these k parameters, so we only change these k parameters. As a result, we change the values $x_1, \dots, x_i, \dots, x_k$ to the values $x_i + \lambda \cdot c_i$, while leaving all the values $x_{k+1}, \dots, x_n, \dots, x_N$ intact.

After this step, in a linear approximation, the value of the efficiency changes to $f(x_1, \dots, x_n)$ to

$$\begin{aligned} f(x_1 + \lambda \cdot c_1, \dots, x_k + \lambda \cdot c_k, x_{k+1}, \dots, x_N) &= \\ f(x_1, \dots, x_k, \dots, x_N) + \sum_{i=1}^k \frac{\partial f}{\partial x_i} \cdot (\lambda \cdot c_i) &= \\ f(x_1, \dots, x_n, \dots) + \lambda \cdot \sum_{i=1}^k c_i^2. \end{aligned}$$

So, the corresponding increase in efficiency

$$\Delta_e \stackrel{\text{def}}{=} f(x_1 + \lambda \cdot c_1, \dots, x_k + \lambda \cdot c_k, x_{k+1}, \dots, x_N) - f(x_1, \dots, x_k, \dots, x_N)$$

is equal to

$$\Delta_e = \lambda \cdot \sum_{i=1}^k c_i^2. \quad (2)$$

To compare Δ_s and Δ_e , let us use Zipf's law. We want to compare the values (1) and (2). The difference between these values depends on the values c_i .

It is known that in many practical situations, the decrease of the sorted values c_i with the index i is described by *Zipf's law* $c_i \approx c \cdot \frac{1}{i}$ for some constant c ; see, e.g., [1, 2].

For this distribution, the sum (1) corresponding to large N can be approximately described as an infinite sum

$$\Delta_s \approx \lambda \cdot \sum_{i=1}^{\infty} c_i^2 = \lambda \cdot c^2 \cdot \sum_{i=1}^{\infty} \frac{1}{i^2}.$$

It is known that the sum $\sum_{i=1}^{\infty} \frac{1}{i^2}$ is equal to $\frac{\pi^2}{6} \approx 1.64$ (for reader's convenience, the derivation of this formula is given in the Appendix). Thus, we have

$$\Delta_s = \frac{\pi^2}{6} \cdot \lambda \cdot c^2 \approx 1.64 \cdot \lambda \cdot c^2. \quad (3)$$

In contrast, for the engineering trial-and-error approach, we similarly get

$$\Delta_e = \left(1 + \dots + \frac{1}{k^2}\right) \cdot \lambda \cdot c^2. \quad (4)$$

Resulting comparison. When the engineering approach is applied to only one parameter $k = 1$, then we get

$$\Delta_e = \lambda \cdot c^2, \quad (5)$$

which is only approximately 61% of the scientific increase (3).

For $k = 2$, we get

$$\Delta_e = \left(1 + \frac{1}{4}\right) \cdot \lambda \cdot c^2 = 1.25 \cdot \lambda \cdot c, \quad (4)$$

which is approximately 78% of the optimal improvement (3).

For $k = 3$, we get

$$\Delta_e = \left(1 + \frac{1}{4} + \frac{1}{9}\right) \cdot \lambda \cdot c^2 \approx 1.36 \cdot \lambda \cdot c, \quad (5)$$

which is approximately 83% of the optimal improvement (3).

For $k = 4$, we get

$$\Delta_e = \left(1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}\right) \cdot \lambda \cdot c^2 \approx 1.42 \cdot \lambda \cdot c, \quad (6)$$

which is approximately 89% of the optimal improvement (3).

In all these cases, the use of the scientific approach indeed leads to significantly better increase in performance – more than 50% better for the typical case when only one most influential parameter is considered. Thus, a quantitative analysis indeed confirms the recommendation presented in [3] and [4]: it is important to use scientific results in engineering design.

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References

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A Appendix: Derivation of the Formula

$$\sum_{i=1}^{\infty} \frac{1}{i^2} = \frac{\pi^2}{6}$$

Main idea. It is known that in general, a polynomial

$$P(x) = x^n + a_1 \cdot x^{n-1} + \dots + a_{n-1} \cdot x + a_n$$

has n roots (maybe complex) x_1, \dots, x_n , and, in terms of these roots, this polynomial can be represented as

$$P(x) = \prod_{i=1}^n (x - x_i).$$

In this equality, we assume that the coefficient at the largest degree is equal to 1. If we divide each side of this equality by the product of all the values x_i , then we get a similar equality in which both sides have a coefficient at a free term (not depending on x) equal to 1:

$$Q(x) \stackrel{\text{def}}{=} \frac{P(x)}{a_n} = \prod_{i=1}^n \left(1 - \frac{x}{x_i}\right),$$

where

$$Q(x) = 1 + b_1 \cdot x + \dots + b^n \cdot x^n.$$

Here, the fact that the coefficient at a free is equal to 1 means that $Q(0) = 1$.

Let us apply this idea to the sine function. The sine function $\sin(x)$ can be approximated by a polynomial with arbitrary accuracy: e.g., we can use more and more terms from the Taylor expansion

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots = x - \frac{x^3}{6} + \frac{x^5}{120} - \dots$$

For the sine function, $\sin(0) = 0$, so we cannot directly apply the above idea to this function. However, if we divide $\sin(x)$ by x , then for $x \rightarrow 0$, the ratio has the limit 1. So, we can apply the above formula and conclude that

$$\frac{\sin(x)}{x} = \prod_i \left(1 - \frac{x}{x_i}\right),$$

where x_i are all the roots of the ratio

$$\frac{\sin(x)}{x} = 1 - \frac{x^2}{6} + \frac{x^4}{120} - \dots$$

It is known that $\sin(x) = 0$ when $x = n \cdot \pi$ for an integer n . The root corresponding to $n = 0$ is not the root of the ratio, so what is left are roots $\pm j \cdot \pi$ for $j = 1, 2, \dots$. Substituting these roots into the above formula, we get

$$\frac{\sin(x)}{x} = \prod_{j=1}^{\infty} \left(1 - \frac{x}{j \cdot \pi}\right) \cdot \left(1 - \frac{x}{j \cdot \pi}\right).$$

By using the formula $(1 - a) \cdot (1 + a) = 1 - a^2$, we can transform this product into

$$\frac{\sin(x)}{x} = \prod_{j=1}^{\infty} \left(1 - \frac{x^2}{j^2 \cdot \pi^2}\right).$$

In general, $(1 - a \cdot x^2) \cdot (1 - b \cdot x^2) = 1 - (a + b) \cdot x^2 + o(x^2)$, and, similarly,

$$(1 - a \cdot x^2) \cdot (1 - b \cdot x^2) \cdot \dots = 1 - (a + b + \dots) \cdot x^2 + o(x^2).$$

Thus, if we now expand both sides in Taylor series, we get

$$1 - \frac{x^2}{6} + \frac{x^4}{120} - \dots = 1 - \sum_{j=1}^{\infty} \frac{1}{j^2 \cdot \pi^2} \cdot x^2 + \dots$$

By equating terms proportional to x^2 in both sides of this equality, we get

$$-\frac{1}{6} = -\sum_{j=1}^{\infty} \frac{1}{j^2 \cdot \pi^2}.$$

Multiplying both sides by $-\pi^2$, we get the desired formula

$$\sum_{j=1}^{\infty} \frac{1}{j^2} = \frac{\pi^2}{6}.$$