

Grading that Takes into Account the Need to Learn from Mistakes

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Abstract

It is well known that the best way to learn the new material is to try it, to make mistakes, and to learn from these mistakes. However, the current grading scheme, in which the overall grade is a weighted average of the grades for all the assignments, exams, etc., does not encourage mistakes: any mistake decreases the grade on the corresponding assignment and thus, decreases the overall grade for the class. It is therefore desirable to modify the usual grading scheme, so that it will take into account – and encourage – learning by mistakes. Such a modification is proposed in this paper.

Specifically, we suggest that the overall grade be – as now – the weighted average of the grades corresponding to different parts of the material, but each of these parts-grades is now calculated differently: instead of the weighted average of grades corresponding to different assignments in which this material is tested, we suggest using the *largest* of the grades corresponding to all these assignments.

1 Formulation of the Problem: Current Grading Schemes Do Not Take Into Account the Importance of Learning from Mistakes

Learning from mistakes is important. It is known that, when learning, it is important to fail, to make mistakes and to learn from these mistakes; see, e.g., [1] and references therein.

Current grading schemes do not encourage learning from mistakes.

In the current grading schemes, the overall grade for the class is usually formed as a weighted average of the grades for all the tests, quizzes, homeworks, lab assignments, etc.

Ideally, a student should be able to make a mistake in one of these assignments, learn from it, and do better later. But if this happens, the grade for this assignment goes down, and, as a result, the overall grade goes down.

Problem: how to modify grading so that it would encourage learning from mistakes. A natural question is: how can we modify the existing grading schemes so that the new grading scheme would encourage students to learn from mistakes.

This is the main problem that we address in this paper. We also want to consider additional problems.

First additional problem: a student may be having a bad day. In the usual grading scheme, if a student had a perfect score on both midterm exams, this means – provided, of course, that the exam was well-designed – that the student mastered the corresponding material and was able to successfully apply the corresponding techniques to solving the relevant problems.

After the two midterm exam, comes the final exam that covers exactly the same topics as both midterms. Suppose that this so-far-perfect student had a bad day on the day of the final exam: maybe the student was not feeling well, maybe the student quarrelled with his/her parents. As a result, this student's performance on the final exam was not perfect. Should we penalize the student for having a bad day?

We know that he/she knows the material, but according to the current grading scheme, a not-so-perfect grade on the final exam decreases the student's overall grade.

Second additional problem: the current grading scheme is also detrimental for struggling students. The first additional problem was about a very good student. But the current grading scheme is not perfect for struggling students as well.

Suppose that a student did very well on the first midterm exam, but in the second part of the semester, did not study that much and, as a result, did very poorly on the second midterm exam. Now is the time for the final exam, so the student needs to prepare – and the time for this preparation is limited, since the student needs to also prepare for other final exams as well.

From the viewpoint of getting the maximum amount of knowledge, within the limited amount of time, the student should concentrate on the material corresponding to the second midterm exam – since he has already mastered the first part of the class (the part corresponding to the first midterm exam). However, if he or she does not study the material corresponding to the first midterm exam at all, his grade on this part of the final will decrease – because we all forget. So, to get a better grade, the student reviews this material as well – at the expense of the time needed to study the second part.

Does this additional review of the first part help? Not really: the student has already mastered this material, so he does not need to learn it again. Yes, the student forgot it somewhat by the time of the exam – but he/she will forget again, so by the time this knowledge is needed (whether in the following class or in real life), it will again be somewhat forgotten – and need to be refreshed.

It is desirable to modify the current grading scheme so that it would encourage such students to concentrate on the parts of the material which are the most important for this student.

Third additional problem: how to make sure that the students at least analyze their mistakes. To learn from mistakes, a student must recognize them, analyze them, and learn from them.

Recognizing the mistakes is what the instructor (or a Teaching Assistant) does when grading the exams and/or the corresponding assignments. Then, the graded exams and assignments are given back to the students.

But do all the students really attentively analyze these graded assignments? In our experience, often, a student who got 90 out of 100 points reads all the comments very attentively, to make sure that he or she get a perfect 100 on this topic next time. On the other hand, students who have barely passed this test often do not even bother to read the comments – in the worst case, they immediately throw away the graded exams, in frustration.

This is not a problem of the grading scheme, but still, it is an important problem: how can we make sure that the students at least read the comments?

2 Analysis of the Problems

Dealing with the main problem. As we have already mentioned, if at some point during the class, a student achieved a very good mastery and knowledge of a certain part of the material, then, no matter when this point occurred, after some time – when this material is needed for a future class or in a job – the student will forget a certain portion of this material.

The same portion will be forgotten whether this perfect mastery was shown on the final exam or on one of the intermediate exams. With this in mind, all we want is to make sure that at least at one point this material was mastered. It is OK if this material was mastered at the time of the first midterm exam – even if on the final exam, due to a natural forgetting, the student’s score is not perfect. Similarly, it is OK if a student has mastered this part of the material by the time of the final exam – even if this student struggled with this material at the time of the first midterm.

Our ultimate goal is to make sure that the student mastered all the parts of the material. In these terms, whether a student mastered it from the very beginning and kept perfect scores for all the assignments and tests, or whether a student first struggled, made mistakes, and only then got to the mastery level – this student did what we expect of him, we can be proud of his/her knowledge of this material, so we should give him a perfect grade for this material.

In general, for each part of the material, what matters is what level the student reached at some point – even if later on, he/she got a worse grade, be it via forgetting or because of having a bad day. In our opinion, a student should not be penalized if at some point, his/her grade for this part of the material was worse. In other words, the grade for each part of the material should not be a *weighted average* of the grades – as it is now – but rather the *maximum* of the grades corresponding to different tests.

Resulting arrangement. So, we arrive at the following modification of the grading scheme: the grade for each piece of the material should be estimated as the maximum of the grades for this material on all the tests that check this part of the material.

For example, if for some material:

- students are first tested by several quizzes, and
- then they have a midterm exam that combines the material from all these quizzes,

then, instead of taking the weighted average of the grade on the quizzes and the grade on the exam, it makes sense to consider the largest of the two grades.

This will allow students to make mistakes and still gain a perfect grade – and thus, this arrangement solves the main problem.

This arrangement also solves the first additional problem. The above arrangement also solves the first additional problem – a perfect student who happened to have a bad day on the day of the midterm exam still have a chance to get a perfect grade for this part of the material, the grade that reflects his/her mastery of this material.

Dealing with the second additional problem. Let us now consider the second additional problem: that in the current grading scheme, a struggling student, who should concentrate on the parts of the material that he does not know well is instead encouraged to also refresh his knowledge of the parts that he/she has already mastered.

This additional problem is also dealt with in the above arrangement: if a student already got a perfect score on the first midterm exam, there is no need for this student to study the corresponding material – his/her final grade for this material will be perfect anyway.

Here is an additional thought: if the overall grade for this part of the material does not depend on how well the student answers the corresponding questions on the final exam, why ask these questions at all? A more productive use of the student's time would be to allow the student to only answer the questions for the part of the material in which he/she has not previously shown mastery.

Dealing with the third additional problem. How to make sure that the students at least read the comments? One possible idea is to use the experience of the final exams.

In the US, students do not get back their final exams, the instructor is supposed to keep the graded final exams for a year. A student, however, can come and look at the final exam, and ask the instructor to explain what exactly went wrong – such questions are strongly encouraged. A natural idea is to do the same for all the exams and assignments. After each exam, as usual, it make sense to describe the correct solutions and to explain typical mistakes. But it is also reasonable to ask students to come and look at their exams – and to ask questions if not everything was perfect and the student is now sure what exactly.

Summarizing, we arrive at the following modified grading scheme.

3 Proposed Modification of the Grading Scheme

Current grading scheme: a reminder. In the current grading scheme, the overall grade is a weighted average of the grades on all the assignments:

$$g = \sum_{i=1}^n w_i \cdot g_i,$$

where n is the overall number of assignments, $g_i \in [0, 100]$ is the grade given to the i -th assignment, and the value $w_i > 0$ are the appropriate weights, for which $\sum_{i=1}^n w_i = 1$.

The grade g_i on each assignment is itself the weighted sum of the grades $g_{ij} \in [0, 100]$ corresponding to different parts j of the material:

$$g_i = \sum_{j \in J_i} w_{ij} \cdot g_{ij}.$$

Here, we denote the number of different parts of the material by m , $J_i \subseteq \{1, \dots, m\}$ is the set of all the topics covered in the i -th assignment, and $w_{ij} > 0$ are the appropriate weights, for which $\sum_{j \in J_i} w_{ij} = 1$ for all i .

Substituting the expressions for g_i in terms of the grades g_{ij} into the formula for the overall grade, we conclude that

$$g = \sum_{i=1}^n w_i \cdot \left(\sum_{j \in J_i} w_{ij} \cdot g_{ij} \right).$$

This sum can be represented as the linear combination of the grades corresponding to different parts of the material:

$$g = \sum_{j=1}^m \sum_{i: j \in J_i} w_{ij} \cdot g_{ij}.$$

The largest possible contribution of the j -th part of the material into the overall grade is attained when all the corresponding grades g_{ij} are perfect:

$$g_{1j} = g_{2j} = \dots = 100.$$

In this case, this contribution is equal to $v_j \cdot 100$, where we denoted

$$v_j \stackrel{\text{def}}{=} \sum_{i:j \in J_i} w_{ij}.$$

In general, the overall grade can therefore be described as

$$g = \sum_{j=1}^m v_j \cdot g'_j,$$

where

$$g'_j = \sum_{i:j \in J_i} \frac{w_{ij}}{v_j} \cdot w_{ij}.$$

In other words:

- the overall grade g is a weighted combination of the overall grades g'_j corresponding to different topics $j = 1, \dots, m$, and
- each grade g'_j is a weighted average of the grades w_{ij} for this material in different assignments i .

The proposed modification. The proposed modification is that for each part j of the material, instead of the *weighted average* of all the grades w_{ij} we should consider the *maximum* of all the grades w_{ij} corresponding to different assignments i that test this part of the material.

In other words, we should still have

$$g = \sum_{j=1}^m v_j \cdot g'_j,$$

but for the grades g'_j corresponding to different parts of the material, we should use a different formula

$$g'_j = \max_{i:j \in J_i} g_{ij}.$$

This modification takes into account the need for students to learn from their mistakes.

Comments. In particular, the new grading arrangement means that if in one of the assignments i , a student has already shown his/her mastery in some part j of the material, i.e., if $g_{ij} \approx 100$, then there is no need for the student to solve the corresponding problems in following assignments: the maximum g'_j will still be high.

This should be explicitly explained to the students, so that, in the situation where a student has a limited time to prepare for the exam, he/she should not waste time reviewing the somewhat forgotten material that he/she knew, but rather concentrate on the topics in which this student was never good before.

And, of course, to solve the third problem – of making sure that the students are aware of their mistakes – it is a good idea not to distribute the graded assignments, but to show them to students instead, encouraging them to inquire about their mistakes.

Our preliminary experience. One of us (F.Z.) implemented these ideas in his teaching, and preliminary results seem to indicate that this has improved the overall level of knowledge.

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References

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