Why 3-D Space? Why 10-D Space? A Possible Simple Geometric Explanation

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Abstract

In physics, the number of observed spatial dimensions (three) is usually taken as an empirical fact, without a deep theoretical explanation. In this paper, we provide a possible simple geometric explanation for the 3-D character of the proper space. We also provide a simple geometric explanation for the number of additional spatial dimensions that some physical theories use. Specifically, it is known that for some physical quantities, the 3-D space model with point-wise particles leads to meaningless infinities. To avoid these infinities, physicists have proposed that particles are more adequately described not as 0-D points, but rather as 1-D strings or, more generally, as multi-D "M-branes". In the corresponding *M-theory*, proper space is 10-dimensional. We provide a possible geometric explanation for the 10-D character of the corresponding space.

1 Why 3-D Space?

Formulation of the problem. Empirically, our space is 3-dimensional: we need three coordinates to uniquely determine each spatial location. Why three and not two or five?

Modern physics mostly takes the number of dimensions for granted, as an empirical fact, but it would nice to come up with a theoretical explanation for this number. The main objective of this paper is to provide such an explanation.

Main idea and the resulting explanation. In classical physics, the world consists of particles.

Particles interact: e.g., positively and negatively charged particles are attracted to each other. However, this does not necessarily mean that we have to go beyond the particles model: in modern physics, interaction between particles is explained as an exchange of the auxiliary particles responsible for this interaction. For example, electromagnetic forces are explained as an exchange of photons – quanta of the electromagnetic field; see, e.g., [1].

With time, particles move in space; thus, each particle forms a 1-D trajectory in space. Particles can collide; one particle can turn into several others, etc. Thus, these trajectories can intersect. So, from the topological viewpoint, trajectories form a *graph*, with trajectories as edges and intersections of trajectories as vertices.

From the *physical* viewpoint, the only meaningful spatial locations are points on this graph. However, from the *mathematical* and *computational* viewpoint, analyzing graphs is difficult, it is easier to analyze multi-D manifolds. Thus, it is convenient to embed the graph into a higher-D space. This is similar to the fact that, from the computational viewpoint, it is easier to consider a solid body as a continuous medium instead of explicitly taking into account its discrete atom-by-atom character; see, e.g., [1].

What is the smallest dimension for which we can embed any graph into the manifold of the corresponding dimension? Clearly, the corresponding space cannot be 2-dimensional:

- while some graphs can be embedded into a plane,
- it is well known that not every graph can be embedded into a plane without creating a non-physical additional intersection.

For example, a graph with 5 vertices all of which are connected to each other cannot be thus embedded; see, e.g., [6].

However, it is also known that every finite graph can be embedded into a 3-D space without creating unnecessary intersections. This may be an explanation of why the usual physical space is 3-dimensional: this is a simplest model containing the actual graph-like space.

2 Beyond Point Particle: Why 10-D Space?

Need to go beyond point particles. At first glance, the classical model of point-wise particle is a good consistent description of the physical Universe. However, a more detailed analysis shows that in this seemingly natural model, when we try to estimate the values of some reasonable physical quantities, we get meaningless infinities.

Indeed, let us compute the overall energy of the electric field of a single point-wise charged particle with charge q. The energy density ρ is known to be proportional to the square of the electric field E: $\rho = c \cdot E^2$ for some constant c. According to the Coulomb's law, $E = \frac{1}{r^2}$, where r is the distance to the particle. Thus, $\rho = c \cdot E^2 = \frac{c \cdot q^2}{r^4}$. The overall energy ε can be obtained if we integrate this density over all spatial locations; thus,

$$\varepsilon = \int \rho(x)\, dV = \int \frac{c \cdot q^2}{r^4}\, dV.$$

Since the integrated function depends only on r, we can integrate over each sphere of radius r and get $dV = 4 \cdot \pi \cdot r^2\,dr$, thus

$$\varepsilon = \int_0^\infty \frac{c \cdot q^2}{r^4} \cdot 4 \cdot \pi \cdot r^2\,dr = c \cdot q^2 \cdot 4 \cdot \pi \cdot \int_0^\infty \frac{dr}{r^2} = -c \cdot q^2 \cdot 4 \cdot \pi \cdot \Big|_0^\infty \frac{1}{r} = \infty.$$

String etc.: a natural idea. Since point-wise 0-D particles lead to infinities, a natural idea is to assume that particle are higher-dimensional objects: 1-D strings or, more generally, multi-D "M-branes". It turns out that in the corresponding *M-theory*, we can avoid infinities if we consider a 10-D proper space (and 11-D space-time); see, e.g., [2, 8].

A possible simple geometric explanation of 10-D character of proper space. How can we explain this 10-D character without involving complicated math? let us go back to our original idea: that all we have in the world are particles.

The only difference now is that instead of 0-D particles that form 1-D trajectories as they move, now we have at least 1-D particles that, as they move, create 2-D "trajectories".

From the topological viewpoint, the resulting trajectories are already continuous, so there is no topological need to embed them into a higher-dimensional space. However, from the computational viewpoint, it may be beneficial to consider such an embedding if this will allow us to be able to deal with a simpler space – e.g., with a simple Euclidean space instead of the general curved Riemannian one.

It is known – it was originally proven by the Nobelist John Nash – that every Riemannian space can be embedded into an Euclidean space of higher dimension. The bound on this dimension has been significantly improved since Nash's original result. The best estimate so far is that every Riemannian space of dimension n can be embedded into an Euclidean space of dimension

$$N = \frac{n \cdot (n+1)}{2} + n + \max(n, 5);$$

see, e.g., [3, 4, 5, 7]. In particular, for the case $n = 2$ of trajectories formed by 1-D particles (strings), we get

$$N = \frac{2 \cdot (2+1)}{2} + 2 + \max(2, 5) = 3 + 2 + 5 = 10.$$

Thus, we indeed get a possible simple geometric explanation of the 10-D character of proper space in M-theories.

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References

[1] R. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Addison Wesley, Boston, Massachusetts, 2005.

[2] B. Greene, *The Elegant Universe: Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory*, W. W. Norton & Company, New York, 2003.

[3] M. Günther, "On the perturbation problem associated to isometric embeddings of Riemannian manifolds, *Ann. Global Anal. Geom.*, 1989, Vol. 7, pp. 69–77.

[4] M. Güunther, "Zum einbettungsatz von J. Nash", *Math. Nachr.*, 1989, Vol. 144, pp. 165–187.

[5] M. Günther, "Isometric embeddings of Riemannian manifolds", *Proceedings of the International Congress of Mathematicians ICM'1990*, Kyoto, Japan, pp. 1137–1143.

[6] R. J. Trudeau, *Introduction to Graph Theory*, Dover Publ., New York, 1993.

[7] C. Villani, "Théorèmes de plogement(s) isométrique(s) de Nash", *Journal de Mathématiques des Élèves de l'Ecole Normal Supérieure de Lyon*, March 2016.

[8] E. Witten, "Fivebranes and knots", *Quantum Topology*, 2012, Vol. 3, No. 1, pp. 1–137.