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WHY HALF-FREQUENCY IN INTELLIGENT COMPACTION

Outline. To gauge how well vibrating rollers have compacted the road segment, it is reasonable to process the acceleration measured by the attached sensors. Theoretically, we expect the resulting signal to be periodic with the same frequency f with which the roller vibrates – and thus, after a Fourier transform, we expect to observe only frequencies which are multiples of the vibration frequency f.

Surprisingly, often, we also observe a peak at half-frequency $f/2$.

In this paper, we explain this empirical phenomenon: we show that it is a particular case of a spontaneous symmetry violation, and that the general physical theory of such symmetry violations explains why namely half-frequency signals are often observed.

What is intelligent compaction: a brief reminder. When a road is constructed, stiff material (e.g., crushed rocks) in place on top of the soil and then compacted to make it even stiffer. After that, asphalt is placed on top of the compacted material – and compacted again.

The compaction effect is cased by the weight of the roller that moves back and forth on top of the pavement. To increase the effect, it also makes sense to add vertical motion – i.e., to use vibrating rollers. This is not done in the cities, since vibration is uncomfortable for people in nearby buildings, but vibrating rollers are routinely used in the country, outside the cities.

Soils differ in quality; materials which are placed on top of the soil may also be different. So, it is difficult to predict how much compaction is needed in each particular case. At present, the result of compaction is gauged by time-consuming post-compaction measurements.

To save time, it is therefore desirable to gauge the compaction quality in real time – by equipping rollers with sensors and by processing the corresponding measurement results. For example, we can measure the acceleration at different moments of time.

This idea of placing a sensor-connected computational device on top of a roller is known as *intelligent compaction*; see, e.g., [2].

Problem. Vibration is a periodic process, with a certain frequency f. We therefore expect the resulting acceleration to be periodic with the same frequency.

It is known that an arbitrary periodic function can be decomposed into Fourier series, i.e., can be represented as a linear combination of sinusoids with frequencies f, $2f$, $3f$, ...

We therefore expect that we apply a continuous Fourier transform to the signal measured by the accelerometer, we would get peaks only at frequencies f, $2f$, etc. We do observe these peaks, but, strangely enough, we often also observe a peak at half-frequency $f/2$ [2]. Why?

What we plan to do. In this paper, we provide an explanation for this empirical phenomenon.

What we observe is a spontaneous symmetry violation. The behavior of a vibrating roller is periodic with some period T, meaning that if we perform a shift by T – or by any multiple of T – the behavior will remain the same.

If the resulting acceleration was symmetric with respect to all these shifts, this would means that it is periodic with the same period T – and thus, with the same frequency f. In this case, as we have mentioned, after applying Fourier transform to this acceleration, we would have observed only frequencies which are multiples of f: f, $2f$, $3f$, etc.

The fact that we observe a half-frequency means that the original symmetry is violated. So, we have what physicists call *spontaneous symmetry violation*; see, e.g., [1]. Such violations occur, e.g., during phase transitions, when an isotropic liquid gets frozen into non-isotropic solid crystals.

Let us use the physical theory of spontaneous symmetry violations to explain the appearance of the half-frequency. To understand why half-frequencies are observed, let us recall the physicists' analysis of spontaneous symmetry violations.

According to this general physical analysis [1], while it is theoretically possible to go from a symmetric state directly to a state with no symmetries at all, such transitions have very low probability. In general, the fewer symmetries are violated in a symmetry-violating transition, the more probable this transition. Thus, the most probable is the transition to the state in which the largest number of symmetries is preserved.

Let us apply this general idea to our problem. The original symmetries involve shifts by multiples of T. We consider spontaneous symmetry violations, in which some of these symmetries disappear in the new state. So, not all shifts by a multiple of T preserve the new state.

In line with the above physical idea, the most probable symmetry violation is the one that preserves the largest number of the original symmetries (i.e., the largest number of the original shifts).

Let $k \cdot T$ be the smallest shift that still preserves the new state. Here, we can have $k = 2, 3, \ldots$ One can easily see that in this case, possible shifts are multiples of $k \cdot T$. In this case, for each large integer N, out of N original shifts by T, $2T$, $\ldots$, $N \cdot T$, we preserve $\frac{N}{k}$ of them: a shift by $k \cdot T$, a shift by $2 \cdot k \cdot T$, etc. So, the smaller k, the more shifts are preserved. Thus, the above physical idea means that the most probable spontaneous symmetry violation corresponds to the smallest possible value $k = 2$.

The resulting period $2T$ corresponds exactly to half-frequency. Thus, we have indeed explained the appearance of half-frequency in intelligent compaction.

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