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COSMOLOGICAL INFLATION: A SIMPLE QUALITATIVE EXPLANATION

Outline. In this paper, we provide a simple qualitative explanation of the cosmological inflation – a phenomenon that at the beginning of the Universe, its size was exponentially increasing.

Cosmological inflation: a brief reminder. In large-scale, the Universe is reasonably uniform. This observed uniformity poses a problem to cosmology; see, e.g., [3].

Indeed, it is easy to explain the uniformity of gas inside a given volume, where molecules interact with each other, as a result of which, inhomogeneities disappear. However, in cosmology, the Universe's expansion goes so fast that different parts of the Universe have no time to interact with each other: we observe distant quasars billions of years after they shine, so by the time our signal goes back, the Universe may have collapsed again.

To explain the observed uniformity, modern cosmology has to go beyond the usual non-quantum equations, and take into account that right after the Big Bang, quantum effects led to an exponential expansion ("cosmological inflation"); see, e.g., [2]. As a result of this expansion, spatially separated areas of the observed Universe come from the same micro-region – and thus, have similar quantities.

Problem: cosmological inflation is difficult to explain. Physicists like to have commonsense back-of-the-envelope explanations of physical phenomena. Of course, in most cases, common sense can only give us qualitative (at best approximate) predictions. Exact quantitative predictions still need computations – which are often rather complex.

The problem with cosmological inflation is that at present, it follows from complex mathematical equations, but there seems to be no commonsense explanation for this phenomenon.

What we do in this paper. In this paper, we provide a (relatively) simple explanation for the cosmological inflation.

Let us start our explanation: a general description of the world's dynamics. At any given moment of time, the state of the world can be described by the values x_i of all possible physical quantities describing this state.

For example, for a system consisting of n particles, we can describe the state by describing masses, coordinates, and velocities of all these particles. If the particles have electric charges, we also need to describe these charges. To describe a field, we need to describe its intensity at different spatial locations, etc.

Physical equations describe how the state of the world changes with time, i.e., how the values x_i describing this state change with time:

$$\frac{dx_i}{dt} = f_i(x_1, x_2, \ldots).$$

In a short vicinity of each moment of time t_0, the values x_i do not change much: $x_i(t) \approx x_i(t_))$ So, in this vicinity, we can expand the right-hand of the above dynamic equation in Taylor series over $\Delta x_i(t) \stackrel{\text{def}}{=} x_i(t) - x_i(t_0)$ and keep only linear terms in this expansion:

$$\frac{d(\Delta x_i)}{dt} = \frac{dx_i}{dt} = f_i(x_1(t_0) + \Delta x_1(t), x_2(t_0) + \Delta x_2(t), \ldots) \approx$$

$$a_i + b_{i1} \cdot \Delta x_1(t) + b_{i2} \cdot \Delta x_2(t) + \ldots,$$

where $a_i \stackrel{\text{def}}{=} f_i(x_1(t_0), x_2(t_0), \ldots)$ and $b_{ij} \stackrel{\text{def}}{=} \dfrac{\partial f_i}{\partial x_j}$.

In other words, in a small vicinity of each moment of time t_0, the world's dynamics is described by a system of linear differential equations with constant coefficients:

$$\frac{d(\Delta x)}{dt} = a + B\Delta x,$$

where Δx and a are vectors with components Δx_i and a_i and B is a matrix formed by coefficients b_{ij}.

A general solution of such a system is well known. Namely, each state can be described as a linear combination of eigenvectors e_i of the matrix B:

$$\Delta x = s_1 \cdot e_1 + s_2 \cdot e_2 + \ldots$$

Thus, to describe the state of the world, instead of the original variables Δx_i, we can use their linear combinations $s_1, s_2, \ldots$ For each new variables s_i, the dynamics is straightforward:

$$\frac{ds_i(t)}{dt} = c_i + \lambda_i \cdot s_i(t),$$

where c_i is a constant and λ_i is the corresponding eigenvalue. Hence, for the shifted variables

$$s'_i(t) \stackrel{\text{def}}{=} s_i(t) + \frac{c_i}{\lambda_i},$$

we get

$$\frac{ds'_i(t)}{dt} = \lambda_i \cdot s'_i(t)$$

and

$$s'_i(t) = s'_i(t_0) \cdot \exp(\lambda_i \cdot (t - t_0)).$$

Comment. Please note that the eigenvalues may be complex – e.g., oscillations correspond to imaginary eigenvalues $\lambda_i = \mathrm{i} \cdot \omega_i$ for some real value ω_i.

Let us start taking quantum effects into account. First, let us take into account that a transition from classical to quantum physics introduces oscillations; see, e.g. [1]. Namely, a classically stationary state with energy E is described, in quantum physics, by a wave function

$$\psi(x,t) = \exp\left(\mathrm{i} \cdot \frac{E \cdot t}{h}\right) \cdot \psi(x).$$

For short periods of time, these oscillations are much stronger than systematic changes.

As we have mentioned, oscillations correspond to imaginary eigenvalues. Thus, it is reasonable to assume that in the first approximation, all eigenvalues are imaginary:

$$\frac{ds'_i(t)}{dt} = \mathrm{i} \cdot \omega_i \cdot s'_i(t)$$

and

$$s'_i(t) = \exp(\mathrm{i} \cdot \omega_i \cdot (t - t_0)) \cdot s'_i(t_0).$$

In this approximation, the absolute value $|s'_i(t)|$ of the corresponding quantity $s'_i(t)$ does not change: $|s'_i(t)| = |s'_i(t_0)|$.

Taking quantum effects into account (cont-d). Another feature of quantum systems is the uncertainty principle: that we cannot measure the exact values of all the physical quantities [1]. For example, any attempt to measure location will change the particle's momentum, and vice versa. For small moments of time, even time itself cannot be measured with too much accuracy. Since we cannot measure time with accuracy exceeding some threshold Δt, this means that, in effect, we cannot distinguish moments of time separated by time intervals smaller than Δt. In effect, this means that instead of the non-quantum continuous time, we now have a discrete time, with possible time values t_0, $t_0 + \Delta t$, $t_0 + 2 \cdot \Delta t$, etc.

So, instead of the differential equation

$$\frac{ds'_i(t)}{dt} = \lim_{h \to 0} \frac{s'_i(t+h) - s'_i(t)}{h} = \mathrm{i} \cdot \omega_i \cdot s'_i(t),$$

we now have a difference equation

$$\frac{s'_i(t+\Delta t) - s'_i(t)}{\Delta t} = \mathrm{i} \cdot \omega_i \cdot s'_i(t).$$

Now, we are ready for our explanation.

Explanation. The above difference equation implies that

$$s'_i(t+\Delta t) - s'_i(t) = \mathrm{i} \cdot \omega_i \cdot \Delta t \cdot s'_i(t),$$

hence

$$s'_i(t+\Delta t) = (1 + \mathrm{i} \cdot \omega_i \cdot \Delta t) \cdot s'_i(t).$$

In terms of absolute values, we now have

$$|s'_i(t+\Delta t)| = |1 + \mathrm{i} \cdot \omega_i \cdot \Delta t| \cdot |s'_i(t)| = q \cdot |s'_i(t)|,$$

where we denoted $q \stackrel{\text{def}}{=} \sqrt{1 + \omega_i^2 \cdot (\Delta t)^2} > 1$.

By induction, we can thus conclude that

$$|s'_i(t + k \cdot \Delta t)| = q^k \cdot |s'_i(t_0)|,$$

i.e., that for moments of time commeasurable with the time quantum Δt, we indeed get exponential growth – which is exactly what cosmological inflation is about.

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Литература.

1. *Feynman, R., Leighton, R., and Sands, M.* The Feynman Lectures on Physics – Boston, Massachusetts: Addison Wesley, 2005.

2. *Guth, A. H.* The Inflationary Universe: The Quest for a New Theory of Cosmic Origins – New York: Basic Books, 1997.

3. *Misner, c. W., Thorne, K. S., and Wheeler, J. A.* Gravitation – San Francisco, California: Freeman Publ., 1973.