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VON NEUMANN-MORGENSTERN SOLUTIONS, QUANTUM PHYSICS, AND STORED PROGRAMS VS. DATA: UNITY OF VON NEUMANN'S LEGACY

Outline. In this paper, we show that several seemingly unrelated topics of John von Neumann's research are actually very closely related.

Von Neumann's legacy. John von Neumann was a versatile researcher; see, e.g., [1,3]. In particular, he founded modern game theory [4], foundations of quantum physics [2], and modern computer architecture.

At first glance, there seems to be no relation between different directions of his research. Somewhat surprisingly, there does not seem to be any deep relation between different directions of John von Neumann's research: it almost looks like several different people performed his research in different directions.

For example, his papers and books on game theory have no mention of any of his research on foundations of quantum physics; his papers and books on computer architecture have no mention of his previous research on foundations of quantum physics and game theory, etc.

But this is strange. This seeming absence of relation sounds strange: the ability to combine various ideas was always one of John von Neumann's strong points. For example, this ability is the main source of his successes in computer architecture: he combined many ideas of different people from different research groups – as well as his own ideas – into a coherent consistent description.

What we plan to do. We plan to show that different directions of von Neumann's research are connected much closer than it seems at first glance. Specifically, we will show that his game-theoretic ideas of core and von Neumann-Morgenstern solution, when properly applied to other research areas, leads to basic notions of these areas. Namely, in quantum physics, we get the notion of the basis in a Hilbert space; in computer applications, we get the notion of data.

Notions of core and von Neumann-Morgenstern solution: a brief reminder. To explain the desired relation between different research areas, let us first briefly recall the notions of core and von Neumann-Morgenstern solution.

These two notions are based on the notion of *dominance*: an outcome y *dominates* an outcome x (denotes $x \to y$) if one of the possible coalitions can force the whole group to move from x to y.

Once we know which outcomes dominate which, which of the outcomes should we choose? A reasonable idea to select an outcome x in such a way that no coalition can change it, i.e., to select an outcome x which is not dominated by any other outcome. The set of all such non-dominated outcomes is known as a *core*.

The problem with this suggestion is that in many situations, there is no core. For example, if we simply divide a given amount of money between several people based on a majority vote, then, no matter what division x we propose, each majority-forming group can force an alternative division y in which this group gets the whole amount and no one else gets anything.

To deal with such situations, John von Neumann and his co-author Oskar Morgenstern proposed the notion of a *social norm* – also known as the *von Neumann-Morgenstern (vNM) solution*. Specifically, a set S of outcomes is a vNM solution if the following two conditions hold:

- first, no two outcomes from the set S dominate each other, and
- second, every outcome which is not in S can be forced into an outcome from S.

The second condition means that we can always force people to obey this social norm, i.e., to select an outcome from the set S. The first condition means that once we have decided to restrict ourselves to outcomes from the set S and selected an outcome from this set, then no coalition can change this decision.

Case of quantum physics: what is a natural analog of dominance? In decision making, we deal with decisions. There are two reasons why decisions change: first, there is a continuous change in outside circumstances that leads to changing values of different decisions. These changes are *not* considered in the classical game theory. The only changes which *are* considered are discrete changes in decision, changes which happen when a coalition decides to change its decision.

In quantum physics, we deal with states. States are usually described as vectors in a Hilbert space. For states, there are also two types of change. There is a continuous change described by Schroedinger's equations: the world changes, and the states change. There is also a possibility of a discrete change: in quantum physics, when we perform a measurement, not only we get the measurement result, but we also usually change the original state. Similarly to the game theory case, let us only consider the discrete changes.

In quantum physics, physical quantities are described by operators in Hilbert space. If we measure a quantity A in a state x, then, as a result, this state changes to one of the eigenvectors y of the operator A, with the probability equal to the $|(x,y)|^2$, where (x,y) denotes the scalar (dot) product of the two states x and y.

When for two states x and y, such a transition $x \to y$ is possible, this means that the corresponding probability $|(x,y)|^2$ is non-zero, i.e., that x and y are *not* orthogonal to each other. Vice versa, if $(x,y) \neq 0$, then for A equal to projection on y, measuring the quantity bA in the state x leads to the state y with a non-zero probability.

Thus, here y dominates x ($x \to y$) if and only if x and y are not orthogonal.

In quantum case, there is no core. Let us analyze what the notions of the core and vNM solution mean for this quantum dominance relation. Let us start with the core.

For every vector x, we can easily find another vector y which is not orthogonal to x. Thus, no vector x is non-dominated, and so the core is empty.

Quantum case: what is the natural analogue of vNM solution? When is a set S of quantum states a vNM solution? The first condition from the above definition of a vNM solution means that no two states from the set S dominate each other. By definition of quantum dominance as non-orthogonality, this simply means that all the vectors from the set S must be orthogonal to each other.

The second condition means that every vector not from S is dominated by one of the vectors from the set S. In other words, if a vector y is not dominated by any vector from S, this vector y must belong to the set S.

For quantum dominance as non-orthogonality, this means that if a vector y is orthogonal to all the vectors from the set S, then it must belong to the set S. This is definitely true if the vectors from S form a *basis* of the set S. Vice versa, if the vectors from S do not form a basic, this means that the linear space L of all their linear combinations does not exhaust all the vectors from the Hilbert space; in this case, any vector which is orthogonal to the space L contradicts the second condition.

So, we conclude that *in the quantum case, a set S is a vNM solution if and only if it is a basis in the Hilbert space.*

Thus, the notion of a basis – one of the main notions of the Hilbert-space approach to quantum physics pioneered by John von Neumann – is indeed a natural analogue of the game-theoretic notion of the von Neumann-Morgenstern solution.

What is a natural analogue of dominance in computer processing? Another area that we would like to consider in this paper is the area of computer architecture and of the corresponding data processing. What are the states here?

In a computer, we store both data and programs. To describe a state of the computer means to describe what data is stored and what programs are stored.

A natural transition $x \to y$ occurs when we run the computer that was originally in the state x. When we run the computer, programs process (and transform) the data, so the state of the computer, in general, changes.

Computer case: what is the analogue of the core? When the state of a computer includes some stored programs, then, in general, running this computer will change the state. So, the only way to have a state which is guaranteed not to change is to only have data.

From this viewpoint, a state x of a computer is not dominated if this state consists only of data. Thus, *in the computer case, a natural analogue of the core is data.*

The distinction between data and stored program was one of the main computer architecture ideas of John von Neumann. It is therefore important that the computer-important notion of data is indeed a natural analogue of the game-theoretic notion of the core.

Conclusion. At first glance, different areas of John von Neumann's research may sound unrelated. However, a rather simple analysis shows a deep relation between his ideas in different research areas. Namely:

- the notion of a basis in the Hilbert space, one of the main ideas behind John von Neumann's formalization of quantum physics, is a direct analogue of the notion of a solution – one of the basic notions of his cooperative game theory; and

- the notion of data, one of the main notions underlying John von Neumann's computer architecture ideas, is a direct analogue of the notion of the core – another basic notion of von Neumann's cooperative game theory.

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