No Idea Is a Bad Idea: A Theoretical Explanation

Christian Servin[1] and Vladik Kreinovich[2]
[1]Computer Science and
Information Technology Systems Department
El Paso Community College
919 Hunter, El Paso, Texas 79915, USA
cservin@gmail.com
[2]Department of Computer Science
University of Texas at El Paso, 500 W. University
El Paso, Texas 79968, USA, vladik@utep.edu

Abstract

Many business publications state that no idea is a bad idea, that even if the idea is, at first glance, not helpful, there are usually some aspects of this idea which are helpful Usually, this statement is based on the experience of the author, and it is given without any theoretical explanation. In this paper, we provide a theoretical explanation for this statement.

1 No Idea Is a Bad Idea

Many business publications state that no idea is a bad idea, that even if the idea is, at first glance, not helpful, there are usually some aspects of this idea which are helpful; see, e.g., [2]. Usually, this statement is based on the experience of the author, and it is given without any theoretical explanation.

In this paper, we provide a theoretical explanation for this statement.

2 Our Explanation

Need for decision making. In many real-life situations, we need to make decisions. Most decisions boil down to selecting values of certain parameters $x_1, \ldots, x_n$. For example:

- when we select an investment portfolio, we select the proportions x_i of different financial instruments (stocks, bonds, etc.) in this portfolio;
- when we decide on an organization's budget, we select amounts allocated to different activities;

- when we decide on a space mission, we select the parameters characterizing the specific mission, such as the mission's useful weight, energy consumption, launch time, mission duration, etc.

According to decision theory, when a rational person makes a decision $x = (x_1, \ldots, x_n)$, this decision tries to maximize a corresponding objective function $f(x_1, \ldots, x_n)$; this function is known as *utility function*; see, e.g., see, e.g., [1, 3, 4, 5].

Our decision making is not perfect. Ideally, we would like to find the alternative x that maximizes the utility function $f(x)$. However, in real life, objective functions are complex, their optimization is not an easy task. As a result, most of our decisions are *sub-optimal*, i.e., can be, in principle, further improved. As a result, ideas for such an improvement are always welcome.

Some ideas are good, some are bad. In terms of the parameters x, an idea means that instead of the values $x_1, \ldots, x_n$ that characterize the current imperfect decision, someone proposes to use a different set of parameters $x'_1, \ldots, x'_n$.

People may be reluctant to immediately switch to the new decision – unless there is a convincing motivation for such a switch. However, it is always possible to start applying the new idea, i.e., to start moving towards the new solution, and see if this improves the decision.

Going in the direction of the new solution means that we start changing each value x_i by an amount proportional to the difference $x'_i - x_i$. In precise terms, this means that we select some small number $\varepsilon > 0$, and we replace the original solution $x = (x_1, \ldots, x_n)$ with a new solution $x''_i = x_i + \varepsilon \cdot (x'_i - x_i)$.

Sometimes, this new solution is good, in the sense that it increases the value of the objective function $f(x'') > f(x)$. If the idea is good, a natural thing is to continue implementing it and thus, enjoys the benefits of this new idea.

However, in other cases, the idea turns out to be bad – in the sense that its implementation makes the situation worse: $f(x'') < f(x)$. A seemingly natural behavior is to abandon this idea and to go back to the original solution x.

Bad ideas may have good aspects. Good news is that we do not have to necessarily accept *all* aspects of the idea. Even ideas which are overall bad may have positive aspects.

Let us show, by using the above formal description of the decision making process, that in the vast majority of cases, this is indeed possible – and thus, every seemingly bad idea can help us improve our original decision.

Which aspects should we choose: analysis of the problem. Suppose that instead of using all aspects of the idea, we only use some of them. To be more precise, instead of changing *all* the values x_i, we only change *some* of these values.

Let $I \subseteq \{1, \ldots, n\}$ denote the set of all the indices i for which we change the values from x_i to $x''_i = x_i + \varepsilon \cdot (x'_i - x_i)$. For indices $i \notin I$, we keep the values x_i unchanged: $x''_i = x_i$.

How will the value of the objective function change? Here, $x''_i = x_i + \Delta x_i$ for some small Δx_i:

- for $i \in I$, we take $\Delta x_i = \varepsilon \cdot (x'_i - x_i)$; while
- for $i \notin I$, we take $\Delta x_i = 0$.

Since the values Δx_i are small, we can expand the expression

$$f(x''_1, \ldots, x''_n) = f(x_1 + \Delta x_1, \ldots, x_n + \Delta x_n)$$

into Taylor series and keep only first order terms in this expansion:

$$f(x''_1, \ldots, x''_n) = f(x_1 + \Delta x_1, \ldots, x_n + \Delta x_n) \approx f(x_1, \ldots, x_n) + \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} \cdot \Delta x_i.$$

Substituting the above expression for Δx_i into this formula, we conclude that

$$f(x''_1, \ldots, x''_n) = f(x_1 + \Delta x_1, \ldots, x_n + \Delta x_n) \approx f(x_1, \ldots, x_n) + \varepsilon \cdot \sum_{i \in I} \frac{\partial f}{\partial x_i} \cdot (x'_i - x_i).$$

So:

- for every index i for which the product $\dfrac{\partial f}{\partial x_i} \cdot \Delta x_i$ is positive, adding this index to the set I increases the value of our objective function, while
- for every index i for which the product $\dfrac{\partial f}{\partial x_i} \cdot \Delta x_i$ is negative, adding this index to the set I decreases the value of our objective function.

Which aspects should we choose: resulting recommendation. To maximally increase the gain, we should:

- take into account all the aspects i of the proposal for which $\dfrac{\partial f}{\partial x_i} \cdot \Delta x_i > 0$, and
- ignore all the aspects i of the proposal for which $\dfrac{\partial f}{\partial x_i} \cdot \Delta x_i < 0$.

So, with the exception of situations in which all the products are negative, we will eb able to use some aspects of the original idea and increase the value of the objective function.

In such situations, even if we start with a bad idea, we can always extract good aspects from this idea and thus, improve our original decision.

How frequent are cases when this will not work? Of course, sometimes we do not get any improvement – when all the products are negative, i.e., when it so happens that for each i, the sign of the proposed change $x'_i - x_i$ is exactly opposite to the sign of the corresponding partial derivative $\dfrac{\partial f}{\partial x_i}$. How frequent are such cases?

If we select a new idea by a random choice, without thinking of how it will affect our solution, then we have the exact same probability 1/2 of selecting plus or minus sign of each deviation Δx_i. The probability that each Δx_i is selected with a wrong sign is 1/2. We make n such selections. Thus, under a natural independence assumption, the probability that we select all n signs wrong – and thus, will not be able to extract any good aspects from the original idea – is equal to 2^{-n}. For complex situations, where n is large, this probability is very small – so in the overwhelming majority of situations, it is possible to extract some good aspects from an overall not-so-good idea.

And this under the assumptions that the new ideas come from random tries, without thinking of how they may affect our system. In practice, people who propose such ideas do think about their effect, so, hopefully, the result will be somewhat better than simply picking an idea at random. Thus, the probability to have an idea which has no good aspects at all is even smaller than 2^{-n} – and is, thus, truly negligible.

We have the desired explanation. Thus, in most cases, with few exceptions, it is possible to select good aspects of the original idea – even if the original idea as a whole was bad. This explains the business statement with which we started this paper.

3 We Can Get Even Better Results

Analysis of the problem. We can achieve even more if instead of simply blindly following or not following different aspects of the original idea, we use the original idea as a starting point of a discussion.

For example, if someone proposes to increase the number of workers at a plant, and this turns out to be a bad idea, we can view this as a start of a discussion about how many workers do we really need at this plant. In this case, if the increase in the number of workers did no help the company, maybe a decrease in the number of workers can be more productive?

In other words, we can follow the same idea of selecting some aspects of the proposal, but this time, we understand the aspects more broadly. For example, in the above case, the aspects of the proposal that we take into account is the proposal to *change* the number of workers — and whether it will be an increase or decrease and how big this increase or decrease will be, is up for discussion.

In this case, we can achieve an even better result: namely, in addition to simply adopting all the aspects i of the original proposal for which the product $\frac{\partial f}{\partial x_i} \cdot \Delta x_i$, we also make changes for those aspects for which this product is negative – by implementing the change in a different direction.

Thus, we arrive at the following recommendation.

Resulting recommendation. For those aspects i of the original proposal x'_i for which the sign of the change $x'_i - x_i$ coincides with the sign of the corresponding partial derivative $\frac{\partial f}{\partial x_i}$, we simply adopt the original proposal, i.e., take

$x_i'' = x_i + \varepsilon \cdot (x_i' - x_i)$.

For all other indices, for which the sign of the proposed change $x_i' - x_i$ is opposite to the sign of the partial derivative $\dfrac{\partial f}{\partial x_i}$, we *reverse* the direction of change, and take $x_i'' = x_i - \varepsilon \cdot (x_i' - x_i)$.

In this case, the gain in the value of the objective function takes the form

$$f(x_1'', \ldots, x_n'') =\approx f(x_1, \ldots, x_n) + \varepsilon \cdot \sum_{i \in I} \left| \frac{\partial f}{\partial x_i} \cdot (x_i' - x_i) \right|.$$

The only case when this does not help is when all the partial derivatives are 0s – i.e, when we are already at a (local) maximum. In all other cases, we get an improvement.

Acknowledgments

This work was supported in part by the National Science Foundation grants HRD-0734825 and HRD-1242122 (Cyber-ShARE Center of Excellence) and DUE-0926721, and by an award "UTEP and Prudential Actuarial Science Academy and Pipeline Initiative" from Prudential Foundation.

References

[1] P. C. Fishburn, *Utility Theory for Decision Making*, John Wiley & Sons Inc., New York, 1969.

[2] D. Garrett, *Diary of a Freedom Writer: The Experience*, Tate Publishing, Mustang, Oklahoma, 2013.

[3] R. D. Luce and R. Raiffa, *Games and Decisions: Introduction and Critical Survey*, Dover, New York, 1989.

[4] H. T. Nguyen, O. Kosheleva, and V. Kreinovich, "Decision making beyond Arrow's 'impossibility theorem', with the analysis of effects of collusion and mutual attraction", *International Journal of Intelligent Systems*, 2009, Vol. 24, No. 1, pp. 27–47.

[5] H. Raiffa, *Decision Analysis*, Addison-Wesley, Reading, Massachusetts, 1970.