Kuznets Curve: A Simple Dynamical System-Based Explanation

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Abstract In the 1950s, a future Nobelist Simon Kuznets discovered the following phenomenon: as a country's economy improves, inequality first grows but then decreases. In this paper, we provide a simple dynamical system-based explanation for this empirical phenomenon.

1 Kuznets Curve: A Brief Reminder and Need for an Explanation

What is the Kuznets curve. In the 1950s, Simon Kuznets, an American economist of Russian origin, showed that as the country's Gross Domestic Product (GDP) increases, inequality first increases and then decreases again [1, 2, 4]. The resulting dependence on inequality on GDP looks like an inverted letter U and is thus called an *inverted U-shaped* dependence or the *Kunzets curve*. For this work, Professor Kuznets was awarded a Nobel Prize in Economics in 1971.

Kuznets curve: a controversy. The Kuznets curve is a purely empirical observation. Economists from different sides of the political spectrum have come up with different (and mutually exclusive) explanations for this empirical fact.

On the one hand, free-market champions use the Kuznets curve as an argument that the governments should not interfere with the free market: inequality will decrease by itself, as soon as the economy improves further. As Ronald Reagan used to say, The rising tide lifts all the boats. Based on this argument, these economists

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recommend that the best way to decrease inequality is to minimize the number of government regulations, and the free market will take care of it.

On the other hand, economists who support the need for government regulations note that while the decrease in inequality may indeed be an empirical fact, in all the developed countries, there was a lot of government intervention, and this intervention is what caused the inequality to decrease. Based on this argument, they recommend that the best way to decrease inequality is to continue with the government regulations.

An additional controversy. It should be mentioned that there is an additional controversy related to the Kuznets curve: namely, some researcher doubt that the Kunzets curve is indeed a universal phenomenon; see, e.g., [3].

What we do in this paper. In this paper, we show that the Kuznets curve phenomenon naturally follows from the general system-based analysis.

2 Analysis of the Problem

Let us describe the phenomenon in precise terms. We start in a situation when the overall economic output is small and therefore, most everyone is poor. In such situations, while there may be a small minority of relatively rich people, most people are poor. In this sense, there is not much inequality.

As the economy grows, people's incomes grow. For each person, his or her income grows until it reaches the level m_i expressing the capability of this person to earn money in the corresponding economy. People are different, so they have somewhat different rates v_i at which they move towards this larger income: some go faster, some go slower.

For simplicity, we can assume that for each person, the rate does not change with time, i.e., that the income of the i-th person income increases at this rate until it reaches the value m_i. At the rate v_i, this takes time $\dfrac{m_i}{v_i}$. So, under this assumption, at each moment of time t, the income $x_i(t)$ of the i-th person is equal to:

- $x_i(t) = v_i \cdot t$ when $t \le \dfrac{m_i}{v_i}$, and
- $x_i(t) = m_i$ for $t \ge \dfrac{m_i}{v_i}$.

The values m_i are centered around the mean $\overline{m}$, with random deviations

$$\Delta m_i \stackrel{\text{def}}{=} m_i - \overline{m}.$$

Similarly, the rates v_i center around the mean $\overline{v}$, with random deviations $\Delta v_i \stackrel{\text{def}}{=} v_i - \overline{v}$.

Since there is no reason to believe that there is a correlation between m_i and v_i, we will assume these variables to be independent.

How can we describe inequality. Perfect equality means that everyone's income is the same. This is equivalent to saying that the standard deviation of income is 0. In general, if the standard deviation is equal to 10% of the average income, then it is reasonable to conclude that we have less inequality that when the standard deviation is equal to 20% of the average income. Thus, a natural measure of inequality is the ratio between the income's standard deviation and its mean value.

Now, we are ready to analyze how inequality changes when the economy improves. Kuznets curve considers three stages:

- the starting stage, when the inequality level is relatively low,
- the intermediate stage, when the level of inequality increases, and
- the final stage, when the level of inequality decreases.

We have already discussed that in the beginning, there is practically no inequality. So, to complete our analysis, we need to consider two other stages: the intermediate stage and the final stage.

What happens on the final stage. Let us start with the final stage, because, as we will see, this stage is easier to analyze. In this final stage, everyone reaches their potential m_i. Thus:

- the average income is equal to the average $\overline{m}$ of the values m_i, and
- the standard deviation is equal to the standard deviation σ_m of the differences Δm_i.

So, on the final stage, the inequality level is equal to the ratio

$$\frac{\sigma_m}{\overline{m}}. \tag{1}$$

What happens on the intermediate stage. In the beginning of the intermediate stage, when few people have reaches their potential m_i, the income of each person is equal to

$$x_i(t) = \frac{m_i}{v_i} \cdot t = \frac{\overline{m} + \Delta m_i}{\overline{v} + \Delta v_i} \cdot t.$$

Here, $\overline{m} + \Delta m_i$ can be represented as $\overline{m} \cdot \left(1 + \dfrac{\Delta m_i}{\overline{m}}\right)$ and similarly, $\overline{v} + \Delta v_i$ can be represented as $\overline{v} \cdot \left(1 + \dfrac{\Delta v_i}{\overline{v}}\right)$. Thus,

$$x_i(t) = \frac{\overline{m}}{\overline{v}} \cdot t \cdot \frac{1 + \dfrac{\Delta m_i}{\overline{m}}}{1 + \dfrac{\Delta v_i}{\overline{v}}}.$$

Differences between different people are, in most cases, not so large, so $|\Delta m_i| \ll \overline{m}$ and $\dfrac{\Delta m_i}{\overline{m}} \ll 1$. Similarly, we can conclude that $|\Delta v_i| \ll \overline{v}$ and thus, $\dfrac{\Delta v_i}{\overline{v}} \ll 1$. Thus, we can expand the above expression for $x_i(t)$ in terms of the small values $\dfrac{\Delta m_i}{\overline{m}}$ and $\dfrac{\Delta v_i}{\overline{v}}$ and keep only linea terms in this expansion. As a result, we get the following formula:

$$x_i(t) = \frac{\overline{m}}{\overline{v}} \cdot t \cdot \left(1 + \frac{\Delta m_i}{\overline{m}} - \frac{\Delta v_i}{\overline{v}}\right).$$

The mean value of Δm_i and Δv_i is 0, so the mean income is equal to

$$\overline{x}(t) = \frac{\overline{m}}{\overline{m}} \cdot t.$$

The standard deviation of Δm_i is equal to σ_m, so the standard deviation of the ratio $\dfrac{\Delta m_i}{\overline{m}}$ is equal to $\dfrac{\sigma_m}{\overline{m}}$. Similarly, the standard deviation of the ratio $\dfrac{\Delta v_i}{\overline{v}}$ is equal to $\dfrac{\sigma_v}{\overline{v}}$.

Since the quantities Δm_i and Δv_i are assumed to be independent, the variance of the expression

$$1 + \frac{\Delta m_i}{\overline{m}} - \frac{\Delta v_i}{\overline{v}} \tag{2}$$

is equal to the sum of the variances of $\dfrac{\Delta m_i}{\overline{m}}$ and $\dfrac{\Delta v_i}{\overline{v}}$. Thus, the corresponding standard deviation is equal to

$$\sqrt{\frac{\sigma_m^2}{(\overline{m})^2} + \frac{\sigma_v^2}{(\overline{v})^2}}.$$

The formula for $x_i(t)$ is obtained by multiplying this expression (2) by a constant $\dfrac{\overline{m}}{\overline{v}} \cdot t$. Thus, the standard deviation $\sigma_x(t)$ can be obtained by multiplying the standard deviation of the above expression (2) by the same constant:

$$\sigma_x(t) = \frac{\overline{m}}{\overline{v}} \cdot t \cdot \sqrt{\frac{\sigma_m^2}{(\overline{m})^2} + \frac{\sigma_v^2}{(\overline{v})^2}}.$$

Dividing this standard deviation y the mean $\overline{x}(t)$, we get the following formula for the inequality level at the intermediate stage:

$$\frac{\sigma_x(t)}{\overline{x}(t)} = \sqrt{\frac{\sigma_m^2}{(\overline{m})^2} + \frac{\sigma_v^2}{(\overline{v})^2}}. \tag{3}$$

Conclusion. By comparing the inequality level (3) at the intermediate stage and the inequality level (1) at the final stage, one can easily see that at the intermediate stage, the inequality is higher:

$$\sqrt{\frac{\sigma_m^2}{(\overline{m})^2}+\frac{\sigma_v^2}{(\overline{v})^2}}>\frac{\sigma_m}{\overline{m}}.$$

This is exactly the Kuznets curve phenomenon.

Thus, we have indeed arrived at a simple justification of the Kuznets curve phenomenon.

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