

# In Education, Delayed Feedback Is Often More Efficient Than Immediate Feedback: A Geometric Explanation

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## Abstract

Feedback is important in education. It is commonly believed that immediate feedback is very important. That is why instructors stay often late at night grading students' assignments – to make sure that the students get their feedback as early as possible. However, surprisingly, experiments show that in many cases, delayed feedback is more efficient than the immediate one. In this paper, we provide a simple geometric explanation of this seemingly counter-intuitive empirical phenomenon.

## 1 Formulation of the Problem

**In education, intermediate feedback is useful.** Empirical data shows that intermediate feedback helps in education. Namely, the very existence of an intermediate test significantly improves the learning outcomes in comparison with the situation when students only learn about their level of knowledge from the final exam; see, e.g., [2].

In [4], we describe a simple geometric model of learning that explains this improvement – and even explains the percentage by which the learning outcomes improve.

**Shall feedback be immediate or somewhat delayed?** Since the feedback provided right after the test is better for learning than the feedback provided only at the end of the class, it seems reasonable to conjecture that the smaller

the delay, the more efficient the feedback. In other words, it seems reasonable to expect that immediate feedback is better than the slightly delayed one.

However, empirical data shows the opposite effect: a feedback with a delay is, in general, more efficient than the immediate feedback; see, e.g., [1, 5]. The effect is not large, it is mostly visible in laboratory-type experiments where all other factors are equalized – and it is not distinguishable in real classroom, when the inevitable differences between the groups mask this effect [3]. However, in the laboratory experiments, a slightly delayed feedback has a small but statistically significant advantage.

**Why delayed feedback is somewhat better: what we do in this paper.**

In this paper, we show that the simple geometric model developed in [4] provides an explanation for this somewhat counter-intuitive fact.

## 2 Our Explanation

**Geometric model: reminder.** The main purpose of teaching is to bring the students from the original knowledge state  $A$  (in which they do not know the class material) to the desired state  $B$  (in which they have a good mastery of this material).

Ideally, the path to knowledge should be the shortest path from  $A$  to  $B$ , the straight line connecting  $A$  and  $B$ . In reality, due to misunderstandings and misconceptions, student deviate from the desired straight line  $AB$  and follow a direction  $AB'$  which is somewhat different from the desired one.

This is where feedback helps: upon receiving feedback, students realize that they had some misconceptions, and thus, start moving towards the desired state  $B$ . In geometric terms, this means that instead of following the segment  $AC$  of the ideal straight line  $AB$ , students first follow a straight line segment  $AC'$  from the original state  $A$  to some point  $C' \neq C$ , and then – after receiving the feedback – a straight line from  $C'$  to  $B$ .

**Problem with immediate feedback.** The instructors' experience enables them to detect small deviations. As a result, the instructor can see even minor differences between the desired direction  $AC$  and the actual direction  $AC'$ .

However, students are not yet that skilled. As a result, they may not understand the difference between the directions as indicated by the instructor. In other words, they may correct the specific things indicated by the instructor, but still do not realize the problems in their understanding that caused them to deviate from the desired path  $AC$  to a slightly different path  $AC'$ .

This is especially true if we follow the above-mentioned empirically supported recommendations and submit frequent intermediate feedback to students. If we submit the feedback at the time when the path  $AC$  followed by the student was reasonably short, then a small deviation of angle  $\alpha$  of the direction  $AC'$  from the desired direction  $AC$  leads to a small deviation of the resulting state of the knowledge  $C'$  from the desired state  $C$ : this deviation is approximately equal to  $AC \cdot \alpha$ . As a result, the distance  $CC' \approx AC \cdot \alpha$  from the actual

state  $C'$  of the student's knowledge to the state  $C$  corresponding to the ideal learning process is so small that a student may not notice the difference between the two states.

**Delayed feedback helps improve the situation.** If we delay the feedback by some time, then, by the time the student receives the feedback, he/she has already followed the original direction  $AC'$  even further, to some point  $D'$ . As a student follows a straight line further than the point  $C'$ , the distance  $DD'$  from the student's actual state  $D'$  and the desired state  $D$  at this moment of time (corresponding to following the perfect learning trajectory  $AB$ ) increases: it is now equal to  $AD' \cdot \alpha$ , where  $AD' \approx AD$  is larger than  $AC' \approx AC$  – and the more we delay, the larger this difference becomes.

As the distance  $DD'$  increases, this distance becomes larger than the student's detection threshold – and thus, a student will clearly see the deviation and therefore correct it.

So, the existence of such a detection threshold explains why the delayed feedback often improves learning.

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