Quantum Ideas in Economics Beyond Quantum Econometrics

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Abstract It is known that computational methods developed for solving equations of quantum physics can be successfully applied to solve economic problems; there is a whole related research area called *quantum econometrics.* Current quantum econometrics techniques are based on a purely mathematical similarity between the corresponding equations, without any attempt to relate the underlying ideas. We believe that the fact that quantum equations can be successfully applied in economics indicates that there is a deeper relation between these areas, beyond a mathematical similarity. In this paper, we show that there is indeed a deep relation between the main ideas of quantum physics and the main ideas behind econometrics.

1 Quantum Ideas in Economics: Why and What Is Known

Why quantum ideas in economics. In most practical problems, once we have a candidate for a solution, we can feasibly check whether this candidate is indeed a solution.

For example, in mathematics, it is often difficult to find a proof of a statement or of its negation. However, once someone produces what intends to be a detailed

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proof, it is feasible for a referee (or even for a computer-based system) to check that all the steps in this text are indeed correct and thus, that the text does indeed constitute a proof.

Similarly, in physics, it is often difficult to find a formula that described the observed phenomena, but once such a formula is proposed, one can feasibly check whether all observations indeed satisfy this formula.

In engineering, it is often difficult to come up with a design that satisfies all the given specifications, but once a design is produced, we can use software packages to check that this design indeed satisfies the specifications. For example, we can check that the designed airplane is indeed stable under allowable winds, that the corresponding stresses do not exceed the prescribed level, etc.

Problems for which we can feasibly check whether a candidate is indeed a solution are known as *problems from the class NP*; see, e.g., [3, 4]. The abbreviation NP stands for *Non-deterministic Polynomial*, where:

- "non-deterministic" means that we are allowed to guess, and
- "polynomial" means that once a guess is produced, the computation time needed to check whether a given guess is a solution should not exceed a polynomial of the length of the input (such polynomial bounds are a formal description of feasibility).

Not all practical problems belong to the class NP:

- For example, if we want to find an *optimal* design, then, in general, it is not easy to check that a given guess is optimal: for that, we would need to compare it with an unfeasible number of all possible designs.
- Similarly, in multi-step conflict situations, it is not easy to check whether a given move is winning or not – checking it would require going over all possible counter-moves of the opposite side.

However, many practical problem are indeed problems from the class NP.

It is still not known whether we can solve all problems from the class NP is feasible (polynomial) time: this is the famous open problem of whether the class NP is equal to the class P of all the problems that can be solved feasibly (i.e., in polynomial time). Most computer scientists believe that NP is different from P.

The fact that we do not know whether NP is different from P means that there is no problem from the class NP for which we have proven that this problem cannot be solved in polynomial time. What *is* proven is that there are problems from the class NP which are as hard as possible within this class, in the sense that every other problem from the class NP can be feasibly reduced to this problem. Such problems are known as *NP-complete.* Many problems of solving non-linear equations (and many other problems) have been proven to be NP-complete.

Historically the first problem for which NP-completeness was proven was the following *propositional satisfiability problem (SAT):*

- given a *propositional formula* F, i.e., a formula obtained from propositional ("yes"-"no") variables v_i by using propositional connectives & (and), $\vee$ (or), and $\neg$ (not),

- find the values of the variables v_i that make the formula F true.

As an illustrative example, we can take $F = (v_1 \vee v_2 \vee \neg v_3) \,\&\, (\neg v_1 \vee v_2)$.

Here, a reduction of a problem A to problem B means that for every instance a of the problem A, we can feasibly compute an appropriate instance b of the problem B for which, once we have a solution to the instance b, we can feasibly transform this solution into a solution to the original instance a.

Let us give a simple example of reduction. The problem of solving an equation $p \cdot x^4 + q \cdot x + r = 0$ can be reduced to the problem of solving a quadratic equation $p \cdot y^2 + q \cdot y + r = 0$. Once we have found a solution y to the quadratic equation, we can find the solutions to the original fourth order equation by computing $x = \pm\sqrt{y}$.

So, once we know that a problem is NP-complete, then any good algorithm for solving this problem automatically becomes a good algorithm for solving all other problems from the class NP. This is not just a theoretical possibility – efficient tools for solving the propositional satisfiability problem (known as *SAT-solvers*) are now used to solve many problems from different application areas.

From this viewpoint, econometrics has many complex problems. Sometimes, we do not have efficient algorithms for solving these problems. In this case, due to the above reduction, it is reasonable to look for other complex (NP-complete) problem, and see if known algorithms for solving these other problems can be used to solve economics-related problems as well.

Where can we find such other problems? Most of the practical problems deal with the physical world. Thus, it is reasonable to look into physics for examples of other complex problems for which efficient algorithms are known.

It is known that adding quantum effects makes problems more complex. Thus, if we look for complex problems in physics, it is reasonable to look for problems of quantum physics. So, we arrive at the idea of trying to see if we can apply known algorithms for solving complex problem of quantum physics to solve complex economics-related problems.

Quantum econometrics: what is known. The idea of using quantum techniques – i.e., techniques for solving quantum equations – to solve economics problems has been successfully implemented. The corresponding techniques are known as *quantum econometrics*. These techniques and their numerous applications are described, e.g., in the seminal book [1].

This book emphasizes that quantum econometrics is based on a *mathematical* similarity of equations, *not* on any similarity between physical ideas of quantum physics and economics ideas.

Our idea and what we do in this paper. The fact that quantum ideas have been very successful in econometric applications makes us think that there may be deeper reasons for the mathematical similarity between the corresponding equations, i.e., that there is indeed some relation between physical ideas of quantum physics and ideas from economics.

In this paper, we show that there is indeed such a relation.

2 Main Ideas Behind Quantum Physics: A Brief Reminder

Need for a reminder. To describe a relation between the main ideas of quantum physics and the main ideas behind econometrics – and to convince the readers that this relation is indeed fundamental, not just a mathematical similarity – let us recall the main ideas behind quantum physics (for more details, see, e.g., [2]).

Quantum physics as physics of micro-world. The main objective of physics is to learn the state of the physical world and to predict its future state. The information about the current state of the physical world comes from measurements. To get the most information about the world, we want to make the measurements as accurate as possible. This means, in particular, that the measurements should disturb the measured object as little as possible – since each such disturbance changes the state of the object.

Traditional physics is the physics of *macro-world*, the physics of objects of macro-size. For such objects, it is usually possible to measure them while disturbing them as little as possible. For example:

- We can measure the distance to an object by sending an ultrasound signal towards the object and measure the time it takes for this signal to get to the object, get reflected, and come back to the sensor.
- We can also perform a similar measurement by sending a laser beam.

In both cases, we can use relatively weak signals, so that the measured object is not affected by this signal.

However, as we study smaller and smaller objects, this becomes more and more complicated. When we send a measuring signal to a body consisting of $\approx 10^{23}$ particles, we can have a relatively very weak signal whose effect on the multi-particle body of interest is small. However, the situation drastically changes if we consider *micro-objects*.

To measure the location of an elementary particle, we need to send another particle – e.g., a photon – to interact with the particle of interest. In this case, the signal that we send is of approximately of the same size as the object itself, and there is thus no way that we can ignore the effect of this signal on the measured object.

In other words, in the micro-world, when we perform a measurement on an object, we change this object. This is one of the main features of the micro-world – known as a the quantum world – that no matter how much we try, we cannot avoid changing the state: whenever we measure the state, we change it.

There is a similar idea in economics. At first glance, economics is a macro-object: when we measure GDP or unemployment, we do not change it, the value remains very accurate. However, econometrics is *not* about measuring different parameters of economics, econometrics is about *discovering new dependencies* that describe the economic data.

From this viewpoint, econometrics has exactly the same effect as quantum physics: once we discover a new dependence, the situation changes.

Indeed, let us consider a simplified example. Suppose that a researcher finds out a better way to predict the price $x(t+2)$ of a certain financial instrument two days from today based on the prices $x(t)$, $y(t)$, $\ldots$, $x(t-1)$, $y(t-1)$, $\ldots$ of this stock and related stocks today and in the previous days.

It is known that the stock values sometimes change drastically. For such change days, based on the newly discovered dependence, we can potentially predict, at day t_0, that the stock value will drastically increase in 2 days, to the level

$$x(t_0+2) \gg x(t_0), x(t_0+1).$$

When we did not know the dependence, this could indeed be a valid prediction:

- the value $x(t_0+1)$ would have equal to what the model predicts based on the prices $x(t_0-1)$, $y(t_0-1)$, $x(t_0-2)$, $y(t_0-2)$, $\ldots$, and
- the value $x(t_0+2)$ would have been equal to what the model predicts based on the prices $x(t_0)$, $y(t_0)$, $\ldots$, $x(t-1)$, $y(t_0-1)$, $\ldots$

However, since we now know the dependence, the traders in the stock exchange know that the price will rise and therefore, will start buying this stock – until its price rises, in day t_0+1, to the level $\rho \cdot x(t_0+2)$, where ρ is a discount that takes into account one day difference (i.e., that takes into account the interest rate that you get in one day by a safe investment like Treasury bonds or bank deposits). This will change the next day's stock price $x(t_0+1)$ from the previously predicted value $x(t_0+1) \ll x(t_0+2)$ to a new value $x(t_0+1) \approx x(t_0+2)$.

So, while the model worked perfectly well until it was discovered, once it is discovered, it longer provides correct predictions – because the stock traders take this model into account when trading and thus, change the dynamics of the system and consequently, modify stock prices.

Similarly, in situations in which the model originally predicted drastic decreases in stock prices, once the model becomes known, it no longer provides accurate predictions; see, e.g., [5].

This is an exact analog of the quantum physics phenomenon:

- In quantum physics, once you learn the value of a quantity describing the object, the actual value of this quantity changes, and the known value is no longer a perfect description of the current state of the particle.
- Similarly, in economics, once we discover the previously unknown dependence between economic quantities, this changes the dynamics of trade and thus, the dependence – which worked well in the past – stops working, at least stops being accurate.

This fundamental similarity may be the reason why techniques for solving quantum equations are so helpful in the economic realm.

Comment. In both cases, the size of the effect depends on the relative size of the object:

- In quantum physics, the effect of measurement on a micro-size body can be minuscule, while for micro-size body, the effect is very drastic.

- Similarly, in economics, if only one person knows the dependence and uses it to buy and sell small amounts of stock, the effect on the stock market will be small. However, nowadays, with financial companies actively investing in data analytics, a dependence uncovered by one researcher cannot be kept secret for long: it will inevitably (and very soon) be discovered by others as well. Once this happens, the effect on the stock market will become large – and it will invalidate the original dependence.

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