Is It Legitimate Statistics or Is It Sexism: Why Discrimination Is Not Rational

Martha Osegueda Escobar, Vladik Kreinovich, and Thach N. Nguyen

Abstract While in the ideal world, everyone should have the same chance to succeed in a given profession, in reality, often the probability of success is different for people of different gender and/or ethnicity. For example, in the US, the probability of a female undergraduate student in computer science to get a PhD is lower than a similar probability for a male student. At first glance, it may seem that in such a situation, if we try to maximize our gain and we have a limited amount of resources, it is reasonable to concentrate on students with the higher probability of success – i.e., on males, and only moral considerations prevent us from pursuing this seemingly economically optimal discriminatory strategy. In this paper, we show that this first impression is wrong: the discriminatory strategy is not only morally wrong, it is also not optimal – and the morally preferable inclusive strategy is actually also economically better.

1 Is It Legitimate Statistics or Is It Sexism?

There are statistical differences. For different reasons, people of different gender and/or different ethnicity have different success rates in different disciplines.

For example, while there are many highly successful female computer scientists, in the US, the percentage of undergraduate female computer science students who go to to eventually defend a PhD is lower than for similar male students – while in some other disciplines (and in some other countries), the difference is reverse.

Martha Osegueda Escobar and Vladik Kreinovich
Department of Computer Science, University of Texas at El Paso, 500 W. University,
El Paso, Texas 79968, USA, e-mail: mcoseguedaescobar@miners.utep.edu, vladik@utep.edu

Thach N. Nguyen
Banking University of Ho Chi Minh City, 56 Hoang Dieu 2, Quan Thu Duc, Thu Duc
Ho Ch Minh City, Vietnam, e-mail: Thachnn@buh.edu.vn

Similarly with ethnicity: for example, the corresponding percentage is higher among Asian-American students than among white students.

An important and difficult challenge. The very fact that the percentage of successful females strongly varies from country to country – even for countries with similar ethnicity – shows that the reasons for the statistical differences are not biological. We need to learn from the success of other countries and other disciplines to make sure that everyone has an equal chance to succeed. This idea may sound straightforward, but in reality, how to do it is an important and difficult challenge, way beyond the scope of this paper.

The problem that we deal in this paper. In this paper, we deal with a more mundane problem: what is the best strategy in the current situation?

A seemingly rational strategy. The situation is very simple and straightforward. We want to graduate a certain number of PhDs. We have limited resources. So, at first glance, it seems that a rational strategy is to concentrate on undergraduate students for whom the probability of success is higher – i.e., on male students – and ignore the female students (since for them, the probability of success is lower).

This argument has nothing to do with prejudice against females: if in a few years, the situation reverses, and the probability of a female student succeeding becomes higher than for male ones, a person following this rational will start concentrating on promising female students only and ignore male students completely.

A similar argument can be applied to hiring: female applicants tend to have a higher probability of retiring early because of their family obligations, so should we stop hiring them? Should we just ignore resumes coming from female applicants and only hire males?

The resulting discriminatory strategy strategy may sound rational, but is it moral? The usual argument against the above hypothetical strategy is that while it may sound rational, it goes against the basic moral principles. Everyone should get a chance to succeed, we should judge every person based on his/her individuality, not based on their gender, race, ethnicity, etc.

This is an explanation many people give; see, e.g., [1].

What we show in this paper. In this paper, a detailed analysis reveals that discriminatory strategies are not just immoral, they are actually *not rational*.

2 Why Discrimination Is Not Rational

Let us start analyzing the problem. Without losing generality, let us consider the problem of hiring. The same argument can be used for selecting the most promising students to “groom” them for graduate school, etc.

For simplicity, let us assume that the candidates belong to two possible groups:

- a group for which the probability of success p is higher; for simplicity, we will call this group *majority* (we say "for simplicity", since in computer science, males are actually a majority, but, e.g., Asian-Americans are not), and
- a group for which the probability of success p' is somewhat lower: $p' < p$; for simplicity, we will call this group *minority*.

The probabilities p and p' can be estimated as the proportion of those who succeeded in each group.

We will compare two strategies. Let us consider two possible strategies:

- a *discriminatory* strategy, in which we ignore all minority applicants and only consider more-probable-to-succeed majority applicants. and
- an *inclusive* strategy, in which we consider all applicants.

We plan to analyze these two strategies from a purely rational, purely economic viewpoint: which one brings more benefit to the company.

From this viewpoint, each of these two strategies has its gains and its losses:

- in the discriminatory strategy, we save some money on analyzing minority applicants, but we miss potential gains that we could have if we hired good female employees;
- in the inclusive strategy, we lose some money on checking the applications of all minority applicants, but we may gain by hiring good female employees.

The question is: if we combine these gains and losses, which of the two strategies will turn out to be the most beneficial?

Let us prepare to evaluate gains and losses. The cost of analyzing an application is approximately the same for all candidates. Let us denote this cost by a.

There is also a cost of training a person and supporting this person through the probation period. Let us denote this cost by t.

What can be drastically different is the gain. Like many other things, potential gains are distributed according the *Zipf law* (see, e.g., [2]): if we denote the lifetime gain from hiring the best possible candidate by G, then:

- the gain from hiring the second best candidate is $\frac{G}{2}$,
- the gain from hiring the third best candidate is $\frac{G}{3}$,
- and, in general, the gain from hiring the i-th best candidate is $\frac{G}{i}$.

Case of inclusive strategy. Let us first consider the profit in the case of the inclusive strategy. Let us first count expenses.

The easiest to evaluate are the expenses related to reviewing applications. In the inclusive strategy, we review all $N+N'$ applications. Reviewing each application requires amount a, so overall, we spend the amount $a \cdot (N+N')$ on these reviews.

The next expense item is training. Let us assume that we have k positions that we want to be eventually filled – e.g., in the case of a university, we have k tenured positions.

Since some of the people we hire will not succeed after a probation period, we hire more people to make sure that at the end, we have k successful folks. In general, from N majority candidates, $p\cdot N$ will succeed if hired. From N' minority candidates, $p'\cdot N'$ will succeed if hired. Overall, if we could hire all of them, we would end up with $p\cdot N+p'\cdot N'$ successful folks. Out of these folks, we select k best. Out of successful folks, the probability of being among the k best is the same, whether it is a successful majority or a successful minority. Thus, out of k best, we will have proportionally many majority and minority folks:

- $k_0\stackrel{\text{def}}{=}k\cdot\dfrac{p\cdot N}{p\cdot N+p'\cdot N'}$ majority folks and
- $k'_0\stackrel{\text{def}}{=}k\cdot\dfrac{p'\cdot N'}{p\cdot N+p'\cdot N'}$ minority folks.

For a majority applicant, the probability of success is p. Thus, to make sure that at the end, we have $k\cdot\dfrac{p\cdot N}{p\cdot N+p'\cdot N'}$ majority employees, we need to hire

$$n_0\stackrel{\text{def}}{=}\frac{k_0}{p}=k\cdot\frac{N}{p\cdot N+p'\cdot N'}$$

majority applicants.

For a minority applicant, the probability of success is p'. Thus, to make sure that at the end, we have $k\cdot\dfrac{p'\cdot N'}{p\cdot N+p'\cdot N'}$ minority employees, we need to hire

$$n'_0\stackrel{\text{def}}{=}\frac{k'_0}{p'}=k\cdot\frac{N'}{p\cdot N+p'\cdot N'}$$

majority applicants.

Overall, we need to hire $n_0+n'_0$ applicants. Training one hire costs the amount t, so the overall expenses on training are equal to

$$t\cdot(n+n_0)=t\cdot\frac{k\cdot(N+N')}{p\cdot N+p'\cdot N'}.$$

Let us now count the gains. Since we considered all the applicants, we are sure that the k folks that remain after the probation period are the k best ones. The best of these folks brings the gain G, the second best brings the gain $\dfrac{G}{2}$, the third best the gain $\dfrac{G}{3}$, etc., until we reach the k-th person who contributes the gain $\dfrac{G}{k}$. The overall gain from all these folks is

$$G+\frac{G}{2}+\frac{G}{3}+\ldots+\frac{G}{k}=G\cdot\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{k}\right).$$

The sum in parentheses is an integral sum for the interval

$$\int_0^k \frac{1}{x}\,dx = \ln(x)|_1^k = \ln(k),$$

so the above sum is approximately equal to this integral. Thus, the gain is equal to $G \cdot \ln(k)$.

Subtracting the expenses from this gain, we conclude that if we use the inclusive hiring strategy, our overall profit is equal to

$$G \cdot \ln(k) - a \cdot (N + N') - t \cdot \frac{k \cdot (N + N')}{p \cdot N + p' \cdot N'}. \tag{1}$$

Case of discriminatory strategy. Let us now consider the case of the discriminatory strategy.

In this case, we only screen N majority candidates, so the amount we spend on screening is $a \cdot N$ (smaller amount that for the inclusive strategy).

We want to end up with k candidates. We only hire majority folks, for whom the probability of success is p. Thus, to end with k employees after the probation period, we need to hire $\dfrac{k}{p}$ folks. The cost of training all these hires is equal to $t \cdot \dfrac{k}{p}$.

What is the gain of all these hires? Out of all $p \cdot N + p' \cdot N'$ potentially successful folks, we hired only the majority persons, i.e., our hiring pool consisted of $p \cdot N$ folks out of $p \cdot N + p' \cdot N'$. The probability p_b that the best of the $p \cdot N + p' \cdot N'$ folks is a majority is thus equal to the proportion of successful majority folks among all successful folks: $p_b = \dfrac{p \cdot N}{p \cdot N + p \cdot N'}$. So, in the formula for the expected gain, the contribution of the best person is not G (as in the case of the inclusive strategy), but rather the product $p_b \cdot G = G \cdot \dfrac{p \cdot N}{p \cdot N + p \cdot N'}$.

Similarly, the probability that the second best person is in the majority (and thus, among the hires) is also equal to p_b. Thus, the contribution of this second best person into the formula for the expected gain is not $\dfrac{G}{2}$, but $p_b \cdot \dfrac{G}{2}$. Same with the 3rd best person, etc.

We need to be careful now as we count further. We end up with k employees, but they are not k best folks, they are k best out of *majority* folks. Overall, there are $p \cdot N$ potentially successful majority folks out of the total amount of $p \cdot N + p' \cdot N'$ successful folks. Thus, when we select k top majority top, there are overall

$$K \stackrel{\text{def}}{=} k \cdot \frac{p \cdot N + p' \cdot N'}{p \cdot N}$$

folks of similar quality. Here, $K = 1 + \dfrac{p' \cdot N'}{p \cdot N}$.

So, in counting down in quality, we have to go down to the K-th person. As a result, the overall gain for this strategy is equal to

$$p_b \cdot G + p_b \cdot \frac{G}{2} + p_b \cdot \frac{G}{3} + \ldots + p_b \cdot G \cdot \frac{G}{K} = p_b \cdot G \cdot \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{K}\right).$$

Here, as before,

$$1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{K} \approx \ln(K),$$

where

$$\ln(K) = \ln\left(k \cdot \left(1 + \frac{p' \cdot N'}{p \cdot N}\right)\right) = \ln(k) + \ln\left(1 + \frac{p' \cdot N'}{p \cdot N}\right).$$

Thus, for the discriminatory strategy, the gain is equal to

$$\frac{p \cdot N}{p \cdot N + p \cdot N'} \cdot G \cdot \left(\ln(k) + \ln\left(1 + \frac{p' \cdot N'}{p \cdot N}\right)\right).$$

By subtracting the expenses from this gain, we conclude that the overall profit of using this strategy is equal to

$$\frac{p \cdot N}{p \cdot N + p \cdot N'} \cdot G \cdot \left(\ln(k) + \ln\left(1 + \frac{p' \cdot N'}{p \cdot N}\right)\right) - a \cdot N - t \cdot \frac{k}{p}. \quad (2)$$

So which of the two strategies is the most profitable? In several realistic numerical examples that we tried, the profit (1) from the inclusive strategy exceeds the profit (2) from the discriminatory strategy.

To have a general result, let us consider the case when what we called “minority” is really a minority, i.e., when the ratio $m \stackrel{\text{def}}{=} \frac{N'}{N}$ is small (so that we can ignore terms which are quadratic or of higher order in terms of m). Let us denote $r \stackrel{\text{def}}{=} \frac{p'}{p} < 1$.

In this case, dividing both numerator and denominator of the training-expenses term in the formula (1), we get

$$t \cdot \frac{k \cdot (N + N')}{p \cdot N + p' \cdot N'} = t \cdot \frac{k(1+m)}{p + p' \cdot m}.$$

Here, $p + p' \cdot m = p \cdot (1 + r \cdot m)$, and

$$\frac{1}{1 + r \cdot m} = 1 - r \cdot m + o(m).$$

Thus, the training-expenses term takes the form

$$t \cdot \frac{k}{p} + t \cdot \frac{k}{p} \cdot (1 - r) \cdot m.$$

So, the overall profit from using the inclusive strategy has the form

$$G \cdot \ln(k) - a \cdot N - t \cdot \frac{k}{p} - a \cdot N \cdot m - t \cdot \frac{k}{p} \cdot (1-r) \cdot m. \tag{1'}$$

Similarly, terms in the formula (2) take the following form:

$$\frac{p \cdot N}{p \cdot N + p' \cdot N'} = \frac{1}{1 + m \cdot r} \approx 1 - m \cdot r$$

and

$$\ln\left(1 + \frac{p' \cdot N'}{p \cdot N}\right) = \ln(1 + r \cdot m) \approx r \cdot m.$$

Thus, the formula (2) takes the following form:

$$G \cdot \ln(k) - a \cdot N - t \cdot \frac{k}{p} - G \cdot \ln(k) \cdot r \cdot m + G \cdot r \cdot m. \tag{2'}$$

So, in comparison with the identical expressions corresponding to $m = 0$, we lose the following amounts proportional to m:

- in the discriminatory case, we lose the amount proportional to $G \cdot (\ln(k) - 1)$, while
- in the inclusive case, we lose the amount proportional to $a \cdot N + t \cdot \frac{k}{p} \cdot (1-r)$.

To compare these losses, we need to take into account that even for the weakest of the k hires, for whom the gain is equal to $\frac{G}{k}$, this gain is still much larger than all the expenses on selection and training – otherwise, the company would not be hiring this person in the first place. The expenses of selecting a person are equal to a, the expenses of training $\frac{1}{p}$ persons (needed for one person to succeed) are $t \cdot \frac{1}{p}$. Thus, we conclude that

$$\frac{G}{k} \gg a + t \cdot \frac{1}{p},$$

hence

$$G \gg a \cdot k + t \cdot \frac{k}{p}$$

thus

$$G \gg a \cdot k + t \cdot \frac{k}{p} \cdot (1-r).$$

Thus, to make sure that the discriminatory-strategy loss is larger than the inclusive-strategy loss, it is sufficient to make sure that

$$G \cdot ((\ln(k) - 1) \cdot r - 1) \geq a \cdot (N - k) :$$

then, by adding the last inequality, we would get the desired one.

This last inequality is definitely true: even the gain $\frac{G}{k}$ of the least productive hire is of the same order as this person's lifetime salary, i.e., in the US, several million dollars, while the cost of scanning all N candidates, even if there is a thousand of them, does not exceed \$100 $\times$ 1000, i.e., 100 000 dollars, which is much smaller than G.

Thus, *the inclusive strategy is indeed economically preferable.*

Comment. Of course, from the purely mathematical viewpoint, there are cases when the discriminatory strategy is more profitable: e.g., when the probability p' of the minority hire's success is close to 0, there is no gain in hiring them, only additional expenses in screening and training. However, in practice, the ratio p'/p is not 0: it can be 0.5, even somewhat less – but still sufficiently positive to make sure that the inclusive strategy is economically preferable.

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References

1. L. Carmichael, S. Stalla-Bourdillon, and S. Taab, "Data mining and automated discrimination: a mixed legal/technical perspective", *IEEE Intelligent Systems*, 2016, Vol. 31, No. 6, pp. 51–55.
2. A. I. Saichev, Y. Malevergne, and D. Sornette, *Theory of Zipf's Law and Beyond*, Springer Verlag, Berlin, Heidelberg, 2010.