Symmetries Are Important

Vladik Kreinovich

Abstract This short article explains why symmetries are important, and how they influenced many research projects in which I participated.

What are symmetries? Why symmetries? Looking back, most of my research has been related to the ideas of symmetry. Why symmetry? And what is symmetry?

Everyone is familiar with symmetry in geometry: if you rotate a ball around its center, the shape of the ball remains the same. Symmetries in physics are similar.

Indeed, how do we gain knowledge? How do we know, for example, that a pen left in the air will fall down with the acceleration of 9.81 meters per square second? We try it once, we try it again, it always falls down. You can shift or rotate, it continues to fall down the same way. So, if we have a new situation and it is similar to the ones in which we observed the pen falling, we predict that the pen will fall in a new situation as well.

At the basis of each prediction is this idea: that we can perform some symmetry transformations like shift or rotation, and the results will not change.

Sometimes the situation is more complex. For example, we observe Ohm's law in one lab, in another lab, etc. – and we conclude that it is universally true.

When mainstream use of symmetries in science started. Because of their importance, symmetries have always been studied by philosophers – and sometimes, they helped scientists as well. However, the mainstream use of symmetries in science started only in the beginning of the 20 century, with Einstein's relativity principle. Relativity principle means that unless we look out of the window, we cannot tell whether we stay or move with a constant velocity.

Einstein did not invent this principle: it was first formulated by Galileo when he travelled on a ship in still waters. But what Einstein did for the first time was used this principle to motivate (and sometimes even derive) exact formulas for physical phenomena. This was his Special Relativity Theory.

Vladik Kreinovich
Department of Computer Science, University of Texas at El Paso, El Paso, TX 79968, USA
e-mail: vladik@utep.edu

And he used another symmetry – that a person in a falling elevator experiences the same weightlessness as an astronaut in space – to motivate his General Relativity Theory; see, e.g., [4, 29].

Symmetries after Einstein. In Special Relativity, in addition to the symmetries, Einstein used many other physical assumptions. Later, it turned out that many of these assumptions were not needed – until my former advisor, a renowned geometer Alexander Danilovich Alexandrov proved in 1949 that the relativity principle is sufficient to derive all the formulas of special relativity [1, 2] (see also [24, 32]).

This was one of the results that started the symmetry revolution in physics. Until then, every new theory was formulated in terms of differential equations. Starting with the quark theory in the early 1960s, physicists rarely propose equations – they propose symmetries, and equations follow from these symmetries [4, 29].

The beginning of my research. When I started working under Alexandrov, I followed in his footsteps. First, I tried to further improve his theorem – e.g., by showing that it remains true even in the realistic case when symmetries are only approximate; see, e.g., [12, 13, 15, 16, 17, 30] and references therein.

But then I started thinking further: OK, new theories can be uniquely determined by their symmetries, what about the old ones? We eventually proved that not only Special Relativity – equations of General Relativity, quantum physics, electrodynamics – all can be derived from the symmetries only, without the need for additional physical assumptions; see, e.g., [7, 8, 14, 18].

Symmetries can also explain phenomena. Symmetries can help not only to explain theories, but to explain phenomena as well.

For example, there are several dozens theories explaining the spiral structure of many galaxies – including our Galaxy. We showed that all possible galactic shapes – and many other physical properties – can be explained via symmetries.

Namely, after the Big Bang, the Universe was uniform. Because of gravity, uniformity is not stable: once you have a part which has slightly higher density, other particles will be attracted to it, and we will have what is called spontaneous symmetry violations. According to statistical physics, violations are most probable when they retain most symmetries – just like when heated, solid body usually first turns into liquid and only then to gas. This explains why first we get a disc, and then a spiral – and then Bode's law, where planets' distances from the Sun form a geometric progression [5, 6, 22].

Symmetries beyond physics. Similarly, symmetries can be helpful in biology – where they explain, e.g., Bertalanfi equations describing growth, in computer science – when they help with testing programs, and in many other disciplines [25].

Symmetries in engineering and data processing. Symmetries not only explain, they can help design.

For example, we used symmetries (including hidden non-geometric ones) to find an optimal design for a network of radiotelescopes [20, 21] – and to come up with optimal algorithms for processing astroimages; see, e.g., [10, 11].

Need for expert knowledge. These applications were a big challenge, because we needed to take into account expert opinions, and these opinions are rarely described in precise terms.

Experts use imprecise linguistic expressions like "small", "close", etc., especially in non-physical areas like biology. Many techniques have been designed for processing such knowledge – these techniques are usually known as fuzzy techniques; see, e.g., [3, 9, 23, 27, 28, 31].

Because of the uncertainty, experts' words allow many interpretations. Some interpretations work better in practice, some do not work so well. Why?

Symmetries help in processing expert knowledge as well. Interestingly, it turned out that natural symmetries can explain which methods of processing expert knowledge work well and which don't; see, e.g., [19, 25, 26].

There are still many challenges ahead. Was it all smooth sailing? Far from it. There are still many important open problems – which is another way of saying that we tried to solve them and failed. And I hope that eventually symmetry ideas can solve them all.

Summarizing. I love symmetries. Physicists, chemists, biologists usually do not need to be convinced: they know that symmetries are one of the major tools in science. Computer scientists also start being convinced.

To the rest: try to find and use symmetries, they may help. And while we are exploring the idea of symmetries, let us look for new exciting ideas that will lead us to an even more exciting future.

Many thanks. I am very grateful for this book. I am grateful to the editors, I am grateful to Springer, and I am grateful to all the authors. I am glad that I have so many talented friends and colleagues.

I myself enjoyed reading the papers from this volume, and I am sure the readers will enjoy reading them too.

Thanks you all!

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