Empirical Power Law for Company Losses: A Probability-Based Explanation

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Formulation of the problem. Companies compete in the market. Both a given company and its competitors constantly develop new products. One of these products becomes a winner – while the efforts of all other companies do not lead to success and thus, qualify as losses.

The more money and efforts the company invests in the development of the new products, the higher the probability that this company will succeed – and vice versa, the fewer money is invested, the higher the probability of failure. Analysis of different companies shows that, on average, the probability of failure $p$ is approximately inversely proportional to the overall investment $I$ in development of new products; to be more precise, $p \approx \frac{c}{I + I_0}$ for some constants $c$ and $I_0$; see, e.g., [1].

The problem is that there is no convincing explanation for the above formula. In this talk, we provide such an explanation.

Comment. Similar dependencies can be found in many application areas. Historically the first such dependence was Zipf’s law – first formulated in linguistics – that states that if the text is large enough, then when we order words in the decreasing order of frequency, the frequency $f_n$ of $n$-th word is approximately equal to $f_n \approx \frac{c}{n + n_0}$ for some constants $c$ and $n_0$. Similar formulas work well when we sort cities in the decreasing order of their population, when we sort companies in the decreasing order of their sizes, papers by number of citations, earthquakes by magnitude, etc.

Our explanation. Let $k$ denote the number of new products developed by a given company. Let $a$ be the average investment need to develop a new product. Then, the overall company’s investment is equal to $I = a \cdot k$. So, in terms of the investment $I$, the value $k$ has the form $k = I/a$.

Let $C$ denote the average number of new products proposed by the competition. Then, the overall number of competing products is $k + C$.

It is reasonable to assume that all these products are equally reasonable and thus, that each of these products has the same probability of becoming a commercial success – probability equal to $1/(k + C)$. The probability that the given company loses can thus be estimated as the probability that one of $C$ competitors’ products will succeed – and is, therefore, equal to $p = C/(k + C)$. Substituting $k = I/a$ into this formula, we get $p = \frac{C}{I/a + C}$. Multiplying both the numerator and the denominator by $a$, we conclude that $p = \frac{a \cdot C}{I + a \cdot C}$.

So, we indeed get the desired expression for $c = I_0 = a \cdot C$.