A Recent Result about Random Metrics Explains Why All of Us Have Similar Learning Potential

Christian Servin[1], Olga Kosheleva[2], and Vladik Kreinovich[3]
[1]Computer Science and
Information Technology Systems Department
El Paso Community College (EPCC)
919 Hunter Dr., El Paso, TX 79915-1908, USA
cservin1@epcc.edu
[2]Department of Teacher Education
[3]Department of Computer Science
University of Texas at El Paso
500 W. University, El Paso, TX 79968, USA
olgak@utep,edu, vladik@utep.edu

Abstract

In the same class, after the same lesson, the amount of learned material often differs drastically, by a factor of ten. Does this mean that people have that different learning abilities? Not really: experiments show that among different students, learning abilities differ by no more than a factor of two. This fact have been successfully used in designing innovative teaching techniques, techniques that help students realize their full learning potential. In this paper, we deal with a different question: how to explain the above experimental result. It turns out that this result about learning abilities – which are, due to genetics, randomly distributed among the human population – can be naturally explained by a recent mathematical result about random metrics.

1 Formulation of the Problem

Learning results differ a lot. In the same class – whether it is an elementary school, a high school, or a university – some students learn a lot right way, and some stay behind and require a lot of time to learn the same concepts.

How can we explain this disparity? There are several possible explanations of the above phenomenon. Crudely speaking, these explanations can be divided into two major categories:

- elitist explanations claim that people are different, some are born geniuses, some have low IQ; nasty elitists claim that the IQ is correlated with gender, race, etc., others believe that geniuses are evenly distributed among people of different gender and different races, but they all believe that some people are much much more capable than others, and this difference explains the observed different in learning results;

- liberal explanations assume that everyone is equal, that every person has approximately the same learning potential, so even the slowest-to-learn students can learn very fast, we just need to find an appropriate teaching approach; see, e.g., [2, 3].

Which explanation is correct: an experiment. To answer this question, Russian specialists in pedagogy made the following experiment – cited in [1, 4, 6, 7]. After a regular class, with the usual widely varying results, the researchers asked the students to write down everything that the students remembered during this class period – what material was taught, what dress the teacher had on, etc., literally everything.

Not surprisingly, straight-A students remembered practically all the material that was being taught, but students whose average grade is on the edge of failing remembered, in the worst case, about 10% of what was taught in the class. This was expected. What was completely unexpected is that when the researchers counted the overall amount of information that a student remembered, the overall number of remembered bits differed by no more than a factor of two. The only difference was that:

- the bits remembered by straight-A students included all the material taught in the class, while
- the bits remembered by not so successful students included how the teacher was dressed, which birds flew outside the window during the class, etc.

This experiment – and other similar experiments (see, e.g., [2] and references therein) – clearly supports what we called a liberal viewpoint.

This experimental result is used in teaching. This observation underlies successful teaching methods – e.g., a method tested and promoted by the Russian researcher Anatoly Zimichev and his group [1, 4, 6, 7] – that we can make everyone learn well if we block all the other sources of outside information (no teacher, online learning, empty room, no other students etc. Other related teaching methods are described in [2, 3].

But why? This experimental results is helpful, but a natural question is: why? How can we explain this results? Why a factor of two and not, e.g., a factor of three or of 1.5?

As we all know, genes are randomly combined, so why don't we have a bigger variety of leaning abilities?

What we do in this paper. In this paper, we provide a possible theoretical explanation for this empirically observed almost-equality.

2 Our Explanation

Let us formulate this problem in precise terms. We are interested in differences between students. A natural way to gauge this difference is to have a numerical value $d(a,b)>0$ that describes how different students a and b are.

Intuitively, the difference between students a and b is exactly the same as the difference between students b and a, so we should have $d(a,b)=d(b,a)$. It is also reasonable to require that the difference $d(a,c)$ between students a and c should not exceed the sum of the differences $d(a,b)$ between a and some other student b and $d(b,c)$ between this auxiliary student b and the original student c: $d(a,c)\le d(a,b)+d(b,c)$. In mathematics, a symmetric ($d(a,b)=d(b,a)$) function satisfying this inequality is known as a *metric*.

We know that genes act randomly, so we expect the metric to also be random – in some reasonable sense. So, we are interested in the properties of a random metric.

Analysis of the problem. We have a large population of people on Earth, the overall number n of people is in billions. So, for all practical purposes, we can assume that this n is close to infinity – in the sense that if there is an asymptotic property of a random metric, then this property is – with high confidence – satisfied for this n.

This assumption makes perfect sense. For example, we know that $1/n$ tends to 0 as n tends to infinity. And indeed, if we divide one slice of pizza into several billion pieces – the favorite elementary school example of division – we will get practically nothing left for each person.

So, we are interested in asymptotic properties of random metrics.

Given students a and b, how can we determine the corresponding distance $d(a,b)$? The only way to do that is by measurement, whether it is counting bits in stories (as in the above example) or in any other way. But measurements are never absolutely accurate: whatever we measure, if we repeat the same procedure one more time, we will get a slightly different result.

- Anyone who had physics labs knows that this is true when we measure any physical quantity, be it weight or current.
- Anyone who had their blood pressure measured at the doctor's office knows that two consequent measurements lead to slightly different results.
- And every psychology student knows that repeating the same test – be it IQ test or any other test – leads to slightly different results.

In other words, as a result of the measurement, we only know the measured quantity with some accuracy.

Let $\varepsilon>0$ denote the accuracy with which we can measure the value $d(a,b)$. This means that, instead of the exact value $d(a,b)$, we, in effect, come up with one of the values 0, ε, 2ε, etc., i.e., the values of the type $k\cdot\varepsilon$ for an integer k. There are finitely many people on Earth, so there are finitely many such differences, so there is the largest of the corresponding values k; let us denote this largest number by r.

The value ε is also not precise defined. If we slightly increase or decrease ε, the value r – which is equal to the ratio between the largest possible distance D and the accuracy ε – correspondingly, slightly decreases or slightly increases. If the original value r was odd, let us slightly decrease ε and get an even number $r+1$ instead. This does not change anything of substance, but helps in the analysis of the problem, since more is known about random metrics for even values r.

Now, we are ready to cite the corresponding mathematical result – that leads to the desired explanation.

Our explanation. A recent result [5] shows that in almost all randomly selected metric spaces, all the distances are between the largest value $D = r \cdot \varepsilon$ and its half $D/2 = (r/2) \cdot \varepsilon$. Here, "almost all" means that as n increases, the probability of this property tends to 1. In view of our comment about asymptotic properties, this means that for humanity as a whole, this property should be true.

Thus, different distances differ by no more than a factor of two – exactly as we observe in the above experiment.

Comment. Strictly speaking, the result from [5] refers to integer-valued metrics, but it can be easily extended to metrics whose values are $k \cdot \varepsilon > 0$ for some number $\varepsilon > 0$.

Acknowledgments

This work was supported in part by the US National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science) and HRD-1242122 (Cyber-ShARE Center of Excellence).

References

[1] R. Aló and O. Kosheleva, "Optimization techniques under uncertain criteria, and their possible use in computerized education", *Proceedings of the 25th International Conference of the North American Fuzzy Information Processing Society NAFIPS'2006*, Montreal, Quebec, Canada, June 3–6, 2006, pp. 593–598.

[2] J. Boaler, *Limitless Mind: Learn, Lead, and Live Without Barriers*, New York, Harper One, 2019.

[3] T. Iuculano et al., "Cognitiv tutoring induces widespread neuroplasticity and remediates brain function in children with mathematical learning disacilities", *Nature Communications*, 2015, Vol. 6, Paper 8453.

[4] L. Longpré, O. Kosheleva, and V. Kreinovich, "How to estimate amount of useful information, in particular under imprecise probability", *Proceedings of the 7th International Workshop on Reliable Engineering Computing REC'2016*, Bochum, Germany, June 15–17, 2016, pp. 257–268.

[5] D. Mubayi and C. Terry, "Discrete metric spaces: structure, enumeration, and 0-1 laws", *The Journal of Symbolic Logic*, 2019, Vol. 84, No. 4, pp. 1293–1325.

[6] A. M. Zimichev, V. Kreinovich, et al. *Automatic control system for psychological experiments*, Final Report on Grant No. 0471-2213-20 from the Soviet Science Foundation, 1982 (in Russian).

[7] A. M. Zimichev, V. Kreinovich, et al. *Automatic system for quick education of computer operators*, Final Report on Grant No. 0471-0212-60 from the Soviet Science Foundation, 1980–82 (in Russian).