How Mathematics and Computing Can Help Fight the Pandemic: Two Pedagogical Examples

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Abstract

With the 2020 pandemic came unexpected mathematical and computational problems. In this paper, we provide two examples of such problems – examples that we present in simplified pedagogical form. The problems are related to the need for social distancing and to the need for fast testing. We hope that these examples will help students better understand the importance of mathematical models.

1 First Example: Need for Social Distancing

Formulation of the problem. This problem is related to the pandemic-related need to observe a social distance of at least 2 meters (6 feet) from each other.

Two persons are on two sides of a narrow-walkway street, waiting for the green light. Then start walking from both sides simultaneously. For simplicity, let us assume that they walk with the same speed.

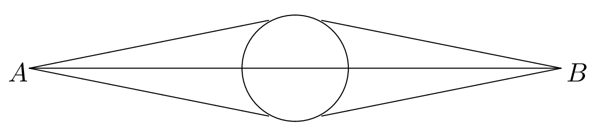
If they follow the shortest distance path – i.e., a straight line connecting their initial locations A and B – they will meet in the middle, which is not good. So one of them should move somewhat to the left, another somewhat to the right. At all moments of time, they should be at least 2 meters away from each other. What is the fastest way for them to do it?

Towards formulating this problem is precise terms. The situation is absolutely symmetric with respect to the reflection in the midpoint M of the segment AB. So, it is reasonable to require that the trajectory of the second person can be obtained from the trajectory of the second person by this reflection. Thus, at any given moment of time, the midpoint M is the midpoint between the two persons. In these terms, the requirement that they are separated by at least 2 meters means that each of them should always be at a distance at least 1 meter from the midpoint M. In other words, both trajectories should avoid the disk of radius 1 meter with a center at the midpoint M.

We want the fastest possible trajectory. Since the speed is assumed to be constant, this means that they should follow the shortest possible trajectory. In other words, we need to find the shortest possible trajectory going from point A to point B that avoids the disk centered at the midpoint M of the segment AB.

Solution. To get the shortest path, outside the disk, the trajectory should be straight, and where it touches the circle, it should be smooth. Thus, the solution is as follows:

- first, we follow a straight line until it touches the circle as a tangent,
- then, we follow the circle,
- and finally, we follow the straight line again – which again starts as a tangent to the circle:



2 Second Example: Need for Fast Testing

Formulation of the problem. One of the challenges related to the COVID-19 pandemic is that this disease has an unusually long incubation period – about 2 weeks. As a result, people with no symptoms may be carrying the virus and infecting others. As of now, the only way to prevent such infection is to perform massive testing of the population. The problem is that there is not enough test kits to test everyone.

What was proposed. To solve this problem, researchers proposed the following idea [1, 2]: instead of testing everyone individually, why not combine material from a group of several people and test each combined sample by using a single test kit. If no viruses are detected in the combined sample, this means that all the people from the corresponding group are virus-free, so there is no need to test them again. After this, we need to individually test only folks from the groups that showed the presence of the virus.

Resulting problem. Suppose that we need to test a large population of N people. Based on the previous testing, we know the proportion p of those who have the virus. In accordance with the above idea, we divide N people into groups. The question is: what should be the size s of each group?

If the size is too small, we are still using too many test kits. If the size is too big, every group, with a high probability, has a sick person, so we are not dismissing any people after such testing, and thus, we are not saving any testing kits at all. So what is the optimal size of the group?

Comment. Of course, this is a simplified formulation, it does not take into account that for large group sizes s, when each individual testing material is diluted too much, tests may not be able to detect infected individuals.

Let us formulate this problem in precise terms. If we divide N people into groups of s persons each, we thus get N/s groups.

The probability that a person is virus-free is $1-p$. Thus, the probability that all s people from a group are virus-free is $(1-p)^s$. So, out of N/s groups, the number of virus-free groups is $(1-p)^s \cdot (N/s)$. Each of these groups has s people, so the overall number of tested people can be obtained by multiplying the number of virus-free groups by s, resulting in $(1-p)^s \cdot N$. For the remaining $N-(1-p)^s \cdot N$ folks, we need individual testing. So, the overall number of needed test kits is

$$N_t = \frac{N}{s} + N - (1-p)^s \cdot N. \tag{1}$$

We want to minimize the number of test kits, i.e., we want to find the group size s for which the number (1) is the smallest possible. Differentiating the expression (1) with respect to s, equating the derivative to 0, and dividing both sides of the resulting equality by N, we get

$$-\frac{1}{s^2} - (1-p)^s \cdot \ln(1-p) = 0. \tag{2}$$

For small p, we have $(1-p)^s \approx 1$ and $\ln(1-p) \approx -p$, so the formula (2) takes the form $-\frac{1}{s^2} + p \approx 0$, i.e.,

$$s \approx \frac{1}{\sqrt{p}}. \tag{3}$$

For example, for $p = 1\%$, we have $s \approx 10$; for $p = 0.1\%$, we get $s \approx 30$; and for $p = 0.01\%$, we get $s \approx 100$.

The resulting number of tests (1) can also be approximately estimated. When the group size s is described by the approximate formula (3), we have $\frac{N}{s} \approx \sqrt{p} \cdot N$. If we take into account that $(1-p)^s \approx 1 - p \cdot s$, then

$$N - (1-p)^s \cdot N \approx p \cdot s \cdot N \approx \sqrt{p} \cdot N.$$

Thus, we get

$$N_t \approx \sqrt{p} \cdot N. \quad (4)$$

For example, for $p = 1\%$, we need 10 times fewer test kits than for individual testing; for $p = 0.1\%$, we need 30 times fewer test kits; and for $p = 0.01\%$, we need 100 times fewer test kits.

Acknowledgments

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References

[1] T. S. Perry, "Researchers are using algorithms to tackle the coronavirus test shortage: the scramble to develop new test kits that deliver faster results", *IEEE Spectrum*, 2020, Vol. 57, No. 6, p. 4.

[2] N. Shental, S. Levy, V. Wuvshet, S. Skorniakov, Y. Shemer-Avni, A. Porgador, and T. Hertz, *Efficient High Throughput SARS-CoV-2 Testing to Detect Asymptomatic Carriers*, medRxiv preprint https://doi.org/10.1101/2020.04.14.20064618, posted on April 20, 2020.