Baudelaire's Ideas of Vagueness and Uniqueness in Art: Algorithm-Based Explanations

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Abstract According to the analysis by the French philosopher Jean-Paul Sartre, the famous French poet and essayist Charles Baudelaire described (and followed) two main – somewhat unusual – ideas about art: that art should be vague, and that to create an object of art, one needs to aim for uniqueness. In this paper, we provide an algorithm-based explanation for these seemingly counter-intuitive ideas, explanation related to Kolmogorov complexity-based formalization of Garrett Birkhoff's theory of beauty.

1 Formulation of the Problem

Baudelaire's ideas about art. In his book [32] about the famous 19 century French poet and essayist Charles Baudelaire, Jean-Paul Sartre emphasizes the following two somewhat unusual aspects of Baudelaire's attitude to art.

The first aspect is explicit in Baudelaire's essays: vagueness. In a well-studied passage of his book *Fusées*, Baudelaire defines beautiful as "Something a little vague, which leaves room for conjecture". This may sound almost trivial now, after the Impressionists changed our understanding of Beauty, but in Baudelaire's time, when beauty was still mostly measured by the Renaissance giants such as Leonardo

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da Vinci or Rafael, with their highly realistic details, this was definitely an almost heretical thought.

The second aspect is not so explicit, but can also be traced to many of his essays and letters: uniqueness, that in order to create someone beautiful, one needs to create something truly unique, repetition is an antithesis of beauty. This also sounds somewhat heretical: there seems to be often a lot of similarity between several beautiful paintings.

A natural question. How can we explain these ideas?

What we do in this paper. In this paper, we show that actually, both seemingly counterintuitive ideas can be explained within a proper algorithm-based formalization of what is beautiful and how can we design a beautiful object.

2 What Is Beauty – Birkhoff's Approach and Its Algorithm-Related Formalization

Bikrhoff's approach. According to the theory developed by the 20th century mathematician Garrett D. Birkhoff – one of the founding fathers of lattice theory – beauty B can be described as the ratio

$$B = \frac{O}{C} \tag{1}$$

between properly defined order O and complexity C; see, e.g., [3, 4, 5, 6, 7, 9]. In the simplest cases, he formalized these notions – and showed that his formula is indeed working.

In his examples:

- Birkhoff defined complexity C as the number of construction steps needed to construct the given object, and
- he defined order as a simplicity of the description: if we can describe an object by using a shorter text, then its order is higher.

Birkhoff's approach reformulated in general algorithmic terms. Birkhoff's theory appeared before the general development of algorithm theory. Now that we are accustomed to the notion of algorithms, it is natural to reformulate his theory in precise algorithmic terms. In these terms, the number of construction steps simply becomes the number of computational steps – i.e., the computation time $t(p)$ of the algorithm p that generates the given object.

The notion of order is a little more difficult to formalize. In his examples, by a description of the objects, Birkhoff meant a complete description, i.e., a description which is detailed enough so that, given this description, we can uniquely reconstruct the object. In other words, the description can serve as a program for a computational device which, given this description, reconstructs the object. In these terms, the length of the description is equal to the length $\ell(p)$ of this program p.

In these terms, the beauty B of an object should be a function of the time $t(p)$ and the length $\ell(p)$ of a program p that generates this object: $B = B(t(p), \ell(p))$. It is well known in computer science that there is a trade-off between the program time and the program length. A short program usually uses only a few ideas of speeding up computations, and thus, takes a reasonable amount of time to run. If we want to speed up the computations, we must add some complicated ideas and modify the algorithm. As a result, to make the program faster, we must usually make it longer. Vice versa, we can often shorten the program by eliminating some of the time-saving parts and thus, by making its running time longer.

In general, if we cut a bit from the program that generates the object x, we get a new program p' which is exactly one bit shorter ($\ell(p') = \ell(p) - 1$). To generate the desired object x, since we do not know whether the deleted bit was 0 or 1, we can try both possible values of this bit (i.e., run two programs $p'0$ and $p'1$) and find out which of the two objects is better. Thus, if we delete a bit, then instead of running the original program p once, we run two programs $p'0$ and $p'1$. Hence, crudely speaking, when we decrease the length of the program by 1, we thus get a double increase in the running time: $t(p') = 2t(p)$.

The new situation is, in effect, the same, the resulting object is the same, the only difference is that we now have $\ell(p') = \ell(p) - 1$ and $t(p') = 2t(p)$. It is therefore reasonable to require that the beauty value $B(t, \ell)$ does not change under this transformation, i.e., that for all possible values of t and ℓ, we have

$$B(t, \ell) = B(2t, \ell - 1). \tag{2}$$

It can be shown (see, e.g., [24]) that every function satisfying this property can be described as a function of the following ratio:

$$r(p) \stackrel{\text{def}}{=} \frac{2^{-\ell(p)}}{t(p)}. \tag{3}$$

Thus, the beauty of the object can be described as the largest possible value of the ratio (2) over all the programs p that generate this object.

Is this an adequate formalization? The ratio (3) is in perfect accordance with Birkhoff's formula (1):

- the time $t(p)$ is exactly what Birkhoff meant by complexity and
- the numerator $2^{-\ell(p)}$ is a decreasing function of the program's (thus description's) length – in perfect accordance with Birkhoff's idea of order.

How this formalization is related to other algorithmic notions. Maximizing the ratio (3) is equivalent to minimizing its inverse $t(p) \cdot 2^{\ell(p)}$ and to minimizing the binary logarithm $\ell(p) + \log_2(t(p))$. From this viewpoint, the beauty of an object is related:

- to the notion of *Kolmogorov complexity* – which is defined as the length of the shortest possible program that generates the given object [28], and

- to resource-bounded versions of Kolmogorov complexity [28] that minimize a combination of the program's length and time.

In this sense, Birkhoff's beauty can be viewed as a particular variant of the resource-bounded Kolmogorov complexity.

3 How This Explains the Need for Vagueness

What is vagueness. Birkhoff's definition is usually applied to abstract objects. However, many objects of art describe real-life objects and/or events: e.g., a painting can reflect a person or a landscape, a poem can describe some events and/or feelings, etc.

Real-life objects can be reproduced with different number of details. For example, we can have a photograph that captures all the details of an object, or we can have a blurred image or even a silhouette, where only some features are reproduced and many details are missing. This is exactly what is meant by vagueness – that some details are missing.

Why is vagueness important for beauty. For each object of art a, we can define its beauty $B(a)$ as the largest possible value of the ratio (3) over all programs that generate this object.

For the same original real-life object x, for reproductions x_v corresponding to different levels of vagueness v, we have, in general different value of beauty $B(x_v)$. If our goal is to make the most beautiful object of art, we should select the level v for which the corresponding beauty $B(x_v)$ is the largest possible.

There are many possible levels; let us denote this number by $L \gg 1$. A priori, we have no reason to assume that one of these levels is more susceptible to beauty: we can enjoy Leonardo's madonnas with lots of detail, and we can enjoy impressionistic painting where most details are missing. Since we do not have any reason to believe that one of these levels is more probable as the most beautiful one, it is reasonable to conclude that each of these levels is equally probable to be the most beautiful one; this reasoning goes back to the 18-19 centuries' mathematician Pierre-Simon Laplace – one of the founders of probability theory – and is therefore known as Laplace's Indeterminacy Principle; see, e.g., [14]. So, each of the L levels has the same probability $1/L$ to be the most beautiful.

In particular, this means that only with probability $1/L \ll 1$, the most beautiful level is the level of all the details. In all other cases, the most beautiful level corresponds to some vagueness – which explains Baudelaire's observation that in the overwhelming majority of cases, vagueness is an important attribute of beauty.

4 Why Uniqueness: An Algorithmic Explanation

We want the most beautiful representation of a real-life object. As we have mentioned earlier, there are many possible representations of an object. Our goal is to select the most beautiful representation.

In abstract terms, our goal is to select a representation a that maximizes the corresponding beauty $B(a)$.

Let us analyze this problem from the algorithmic viewpoint. In contrast to science – that studies objects that already exist – art is about creating new objects. So, it makes sense to think of algorithms that can help in this creation.

Art can reflect everything, so the corresponding optimization problems are very generic. In general, the problem of finding the object that maximizes a given computable function is not algorithmically solvable (see, e.g., [1, 10, 11, 12, 13, 23, 26, 31]), but there is an important case when, under some reasonable condition, the corresponding algorithm *is* possible: the case when there is exactly *one* optimizing object; see, e.g., [15, 16, 17, 18, 19, 20, 23, 25, 27, 29, 30].

Interestingly, if we consider all the cases when there are *two* equally good optimizing objects, such an algorithm is no longer possible; see, e.g., [23, 30]. In this sense, the case of uniqueness is the most general case we can consider if we want our problems to be algorithmically solvable.

Comment. There is also some evidence that even when the algorithms for the multi-optima case are possible, in general, algorithms corresponding to the single-optimum case are more efficient; see, e.g., [2]; see, however, [33].

Conclusion. Thus, if we want to actually *create* a beautiful artistic reflection of a given real-life object or situation, a natural idea is to impose additional restrictions that would make the optimal reflection unique. This is exactly what Sartre described as one of the main Baudelaire's ideas. Thus, this idea is indeed explained.

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