Today is Friday the 13th, the day when vampires, zombies, and other creatures roam the Earth :-(

1-2. Recursion: According to the legend, when a vampire bites a person, this person also becomes a vampire. Let us assume that a vampire bites two people every night, thus creating two new vampires; on the other hand, Buffy the Vampire Slayer slays two vampires every night (if there are any vampires left). As a result, the number \( v(t) \) of vampires on Day \( t \) is equal to the number of vampires \( v(t-1) \) on day \( t-1 \) times 3 minus 2: \( v(t) = \max(3 \times v(t-1) - 2, 0) \). Your task: given the number \( v(0) \) of vampires at day 0, and a non-negative integer \( t \), compute how many vampires there will be at day \( t \).

   a. Write a recursive pseudo-code algorithm that solves this problem.
   b. Transform this algorithm into a recursive method in Java.
   c. Trace your method by predicting the number of vampires in Day 2 for the following two starting numbers of vampires: \( v(0) = 2 \) and \( v(0) = 4 \).
   d. Describe advantages and disadvantages of recursion vs. iteration in solving a problem, from the viewpoint of easiness to write, easiness to understand, and running time.
3. **Multi-Dimensional Arrays:** Let us assume that a knowledge of what weapon helps against what type of creature is stored in a 2-D array. Each row describes a creature and each column a weapon. The corresponding value in each cell is true if this weapon helps against this creature. For example, a wooden stake and bright sun help against vampires, but not against zombies.

a. Write a method for checking whether each creature can be defeated by one of the weapons. For example, for the array
   
   | T  | T | F |
   | F  | F | T |
   
   the answer should be "true", because Creature 0 can be defeated by Weapons 0 and 1, while Creature 1 can be defeated by Weapon 2. On the other hand, if we add an invincible Creature 2 and get an array
   
   | T  | T | F |
   | F  | F | T |
   | F  | F | F |
   
   the answer should be "false".

b. Trace your method on the example of the second array.

c. Using big-oh notation, give the worst-case running time for your algorithm.
4. **Stacks:** To defeat the monsters, good wizards need to come up with a magic number 666. One way to get this number from today's date (05/13/2011) is to compute the value \((13 \times 3 - 2) \times (13 + 5)\) (which is equal to \(37 \times 18 = 666\)).

a. Show, step by step, how by using a stack, we can transform this expression into a postfix form.

b. Show, step by step, how a stack can be used to compute the value of the resulting postfix expression.
5. *Queues:* Witches, vampires, and zombies come to a midnight Friday the 13th party at Area 51, where they form a line while the aliens check their credentials.

a. Show, step by step, the state of this queue, implemented as a 3-element array, when first Witch 1 arrives, then Vampire 1 and Witch 2, then the first two creatures in line pass the checkpoint, and after that Vampire 2 and Zombie 1 arrive.

b. Write a method for dequeuing an element in a queue represented as an array.

c. Using big-oh notation, what is the worst-case running time for your dequeue method?
6-8. **Sorting:** On Friday the 13th five new books were released in the "X for Dummies" series: "Vampires for Dummies", "Zombies for Dummies", "Witches for Dummies", "Mummies for Dummies", and "Aliens for Dummies."

a. Show, step by step, how quicksort will sort the list of titles in alphabetic order. *For extra credit:* also show how insertion sort will do it.
b. Write a Java method implementing quicksort.
c. Draw a table listing the worst-case and average-case complexity for each of these sorting algorithms: insertion sort, selection sort, mergesort, quicksort, and heapsort.
d. True or False: It is possible to implement a sorting algorithm that is much faster than mergesort. Justify your answer.
e. Suppose you start with an empty AVL binary search tree. Show step by step what will happen if you add the above titles to the binary search tree in the original order (V, Z, W, M, and A). Balance at each step where it is necessary.
9. **Search:** The Magic Encyclopedia lists all the knowledge about creatures in alphabetic order. A shortened edition contains entries about only aliens, mummies, vampires, witches, and zombies.

   a. Show, step by step, how sequential search and binary search will look for an entry about witches in this encyclopedia.

   b. What is the worst-case and average-case number complexity of each search?

   c. What are relative advantages and disadvantages of these two search algorithms?

   d. Assuming that the list is implemented as an array, write a Java method implementing sequential search and a Java method implementing binary search.
10. **Hash Tables:**
   a. Show, step by step, how a hash table with five "buckets" will look if we sequentially add to it elements 13, 5, 20, 11, and 666. Assume that as a hash function, we take remainder modulo 5; that is, \( h(n) = n \% 5 \). Use any reasonable way of dealing with the need to place several elements into a single bucket.
   b. What are advantages and disadvantages of hash tables?
   c. List two properties of a good hash function.