In 1908 on this day, May 10, Mother's Day was first celebrated in the US.

1-2. **Recursion:** When Mother's Day starts in a country, after a few years neighboring countries will decide to have Mother's Day as well. For simplicity, let us assume that each country has two new neighbors and that it takes three years for the legislatures to adopt the new laws; so after three years, the number of celebrating countries triples. As a result, the number \( n(t) \) of countries that celebrate Mother's Day in year \( t \) can be computed as \( n(t) = \max(3 \times n(t - 3), n(t - 1)) \). In years \( t < 1908 \) this number was \( n(t) = 0 \). In the year \( t = 1908 \) there is exactly one such country, so \( n(1908) = 1 \). Your task: given a non-negative integer \( t \), compute how many countries celebrate Mother's Day in Year \( t \).

- a. Write a recursive pseudo-code algorithm that solves this problem.
- b. Transform this algorithm into a recursive method in Java.
- c. Trace your method by predicting the number of countries celebrating Mother's Day in year 1912.
- d. Describe advantages and disadvantages of recursion vs. iteration in solving a problem, from the viewpoint of easiness to write, easiness to understand, and running time.
3. Multi-Dimensional Arrays: To save environment, some CS 2401 students plan to share a ride to the mall to buy presents for their moms. Let us assume that their mothers’ tastes are described in a 2-D array. Each row corresponds to one of the mothers, and each column to a possible gift sold at this mall; the corresponding value in each cell describes how happy this mom will be with this gift. This value is either v (very happy), h (happy), or s (somewhat happy).

   a. Write a Java method for checking whether at this mall, it is possible to buy gifts that will make all the moms very happy. For example, for the array

      v  h  s
      s  s  v

      the answer should be "true", since Mom 0 will be very happy to receive Gift 0, while Mom 1 will be very happy to receive Gift 2. However, if we add Mom 3 who does not want any gifts but just wants her kid to get a good grade in CS 2401, we get an array

      v  h  s
      s  s  v
      s  s  s

      for which the answer should be "false".

   b. Trace your method on the example of the second array.

   c. Using big-oh notation, give the worst-case running time for your algorithm.
4. **Stacks:** As a gift for her mother, a student buys three cute teddy bears for $10 each and a bouquet of flowers for $12 (less a $2 student discount). As a result, the amount she pays is $3 \times 10 + (12 - 2)$.

   a. Show, step by step, how by using a stack, we can transform this expression into a postfix form.
   
   b. Show, step by step, how a stack can be used to compute the value of the resulting postfix expression.
5. **Queues:** Several siblings form a line to congratulate their mother with Mother's Day.
   a. Show, step by step, the state of this queue, implemented as a 3-element array, when first Sibling A comes, then B, and C, then the first two siblings have to go back to work, and after that Siblings D and E arrive.
   b. Write a Java method for enqueuing an element in a queue represented as a linked list.
   c. Using big-oh notation, what is the worst-case running time for your enqueue method?
6-8. Sorting: On Mother's Day, five new types of flowers are brought to UTEP Bookstore: roses, anemones, lilies, gladioli, and dahlias.
   a. Show, step by step, how mergesort will sort these flowers in alphabetic order. For extra credit: also show how selection sort will do it.
   b. Write a Java method implementing mergesort.
   c. Draw a table listing the worst-case and average-case complexity for each of these sorting algorithms: insertion sort, selection sort, mergesort, quicksort, and heapsort.
   d. True or False: It is possible to implement a sorting algorithm that is much faster than heapsort. Justify your answer.
   e. Suppose you start with an empty AVL binary search tree. Show step by step what will happen if you add the above flowers to the binary search tree in the original order (roses, anemones, lilies, gladioli, and dahlias). Balance at each step where it is necessary.
9. **Search:** A book about flowers lists all the knowledge about them in alphabetic order. A shortened edition only contains entries about anemones, dahlia, gladioli, lilies, and roses.
   a. Show, step by step, how sequential search and binary search will search for an entry about dahlia in this book.
   b. What is the worst-case and average-case number complexity of each search?
   c. What are relative advantages and disadvantages of these two search algorithms?
   d. Assuming that the list is implemented as an array, write a Java method implementing sequential search and a Java method for implementing binary search.
10. **Hash Tables:**
   a. Show, step by step, how a hash table with five "buckets" will look if we sequentially add to it elements 10, 5, 2, 11, and 24. Assume that as a hash function, we take remainder modulo 5; that is, \( h(n) = n \mod 5 \). Use any reasonable way of dealing with the need to place several elements into a single bucket.
   b. What are advantages and disadvantages of hash tables?
   c. List two properties of a good hash function.