CS 3350 Automata, Computability, and Formal Languages
Fall 2015, Test 1, 3-4:20 pm version

1. Prove that the cubic root of 5 is irrational.

Proof: By Contradiction

Assume $\sqrt[3]{5}$ is rational, then $3\sqrt[3]{5} = \frac{m}{n}$ for some integers $m$ and $n$.

We can always divide both $m$ and $n$ by their common divisors so we can take $m$ and $n$ that have no common divisors.

If we cube both sides we get $5 = \frac{m^3}{n^3}$

Multiplying both sides by the denominator gives: $5n^3 = m^3$

Now the left hand side is divisible by 5 and the right hand side $m^3 = m \times m \times m$ is also divisible by 5 meaning that $m$ is divisible by 5 such that there exist an integer $p$ such that $m = 5p$.

So $5n^3 = m^3$ can be written as

$\Rightarrow 5n^3 = (5p)^3$

$\Rightarrow 5n^3 = 125p^3$

Both sides are divisible by 5

$\Rightarrow n^3 = 25p^3$

and $25p^3$ is divisible by 5

meaning that $n^3 = n \times n \times n$ is divisible by 5

thus $n$ is divisible by 5, so $m$ and $n$ have a common factor. (5)

A contradiction so our assumption was wrong

and $\sqrt[3]{5}$ is not rational.
2-3. Use a general algorithm to design a non-deterministic finite automaton recognizing the language \( L \cup (0^* 1) \). After that, use the general algorithm to design a deterministic finite automaton recognizing this same language.

![Non-deterministic Finite Automaton Diagram](image1)

![Deterministic Finite Automaton Diagram](image2)
4. Use a general algorithm to transform the following finite automaton into the corresponding regular expression. This automaton has two states: the starting state q1 and the final state q2. In the state q1, 1 leads to q2, and 0 leads to q1. In the state q2, 1 leads to q2, and 0 leads to q1.

\[
R_{ij} = R_{ij} U (R_{ik} R_{kj}^* R_{ij})
\]

Regular Expression

\[
R'_{nq_2} = R_{nq_2} U (R_{nq_1} R_{q_1q_2}^* R_{q_2q_1})
\]
\[
\phi U (\lambda 0^* 1) = 0^* 1
\]

\[
R'_{nf} = R_{nf} U (R_{nq_1} R_{q_1q_1}^* R_{nf})
\]
\[
\phi U (\lambda 0^* \phi) = \phi
\]

\[
R'_{q_1q_2} = R_{q_1q_2} U (R_{q_2q_1} R_{q_1q_1}^* R_{q_2q_2})
\]
\[
1 U (0 0^* 1) = 00^* 1
\]

\[
R'_{q_2f} = R_{q_2f} U (R_{q_2q_1} R_{q_1q_1}^* R_{q_2f})
\]
\[
\lambda U (0 0^* \phi) = \lambda
\]

\[
R'_{nf} = 0^* 1 (00^* 1)^*
\]
5. On the example of the automaton from Problem 4, explain, in detail, how the sequence $101010$ will be presented as $xyz$ according to the pumping lemma. For this sequence, check -- by tracing step-by-step -- that the sequence $xy^iz$ for $i = 2$ is indeed accepted by the automaton.

$x =$ anything before the repetition
$y =$ the repetition
$z =$ Everything after repetition

Sequence:

$x$

$q_1\quad q_2\quad q_1\quad q_2\quad q_1\quad q_2$

$y$ = 10

$z$ = 101

for $i = 2 \quad xyy\ z$

$q_1\quad q_2\quad q_1\quad q_2\quad q_1\quad q_2\quad q_1\quad q_2\quad q_2$  Ends in a final state

Therefore $xy^iz$ for $i = 2$

is accepted by the automaton
6. Use the Pumping Lemma to prove that the language $L$ consisting of all three-times repetitions $www$ is not regular.

**Theorem**: $L = \{WWW? \}$ is not regular

$L = \{ 0^p1^p0^p1^p0^p1^p \}$

**Proof**: By Contradiction

Let's assume that $L$ is a regular language. Then according to pumping lemma there exists $p$ such that $\forall s \in L \quad \text{len}(s) \geq p$ there exists $\exists xy^z \text{such that } s = xy^z$

and $\text{len}(xp) \leq p$ and $\text{len}(y) > 0$ and $\forall i \quad x^iy^z \in L$.

Let's take $s = 0^p1^p0^p1^p0^p1^p \in L$

$s = 0_10_20_31_40_51_60_71_80_91_{10}..1_{10}..0_1...0_1...p$

Since $\text{len}(xy) < p$ and the first $p$ symbols are $0$'s, thus $y$ can only consist of $0$'s. By pumping lemma, $i = 2 \quad x^2y^2 \in L$ means that a $y$ is added to $xy$ where adding a $y$ means adding $0$'s; previously in $s$ the number of $0$'s $= \text{number of } L's$, but now since $i = 2$ the number of $0$'s $> \text{number of } L's$ so $x^2y^2 \notin L$.

A contradiction; meaning that our assumption was wrong so $L$ is not regular.