1. Prove that the cubic root of 3 is irrational.

\[ \sqrt[3]{3} \]

Proof by contradiction. Let's assume \( \sqrt[3]{3} \) is rational, meaning \( \sqrt[3]{3} = \frac{m}{n} \) for some integers \( m, n \), where \( m \) and \( n \) have no common divisors.

Cube both sides

\[ \left( \sqrt[3]{3} \right)^3 \cdot \left( \frac{m}{n} \right)^3 \]

\[ 3 = \frac{m^3}{n^3} \]

Get common denominator

\[ m^3 = 3n^3 \]

\( 3n^3 \) is divisible by 3 so \( m^3 \) is divisible by 3

So \( m \) is divisible by 3.

Now, \( m = 3p \) for some integer \( p \)

\[ (3p)^3 = 3n^3 \]

\[ 27p^3 = 3n^3 \]

Divide by 3

\[ 9p^3 = n^3 \]

\( 9p^3 \) is divisible by 3, so \( n^3 \) is divisible by 3.

So \( n \) is divisible by 3.

\( m \) and \( n \) have a common factor 3 so we get a contradiction.

So our assumption is wrong, so \( \sqrt[3]{3} \) is not rational.
2-3. Use a general algorithm to design a non-deterministic finite automaton recognizing the language $1(0^* + 1)$. After that, use the general algorithm to design a deterministic finite automaton recognizing this same language.

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DFA
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3.

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UDFA
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4. Use a general algorithm to transform the following finite automaton into the corresponding regular expression. This automaton has two states: a state q1 which is both a start state and a final state, and a state q2. In the state q1, 0 leads to q2, and 1 leads to q1. In the state q2, 0 leads to q2, and 1 leads to q1.

\[ R_{12} = R_{12} \cup R_{13} R_{23}^* R_{32} \]
\[ \Lambda \cup \emptyset \cup \emptyset \cup \emptyset = \Lambda \]
\[ R_{22} = R_{22} \cup R_{23} R_{33}^* R_{32} \]
\[ \Lambda \cup \emptyset \cup \emptyset \cup \emptyset = 1000^* \]
\[ R_{14} = R_{14} \cup R_{13} R_{33}^* R_{34} \]
\[ \emptyset \cup \emptyset = \emptyset \]
\[ R_{24} = R_{24} \cup R_{23} R_{33}^* R_{34} \]
\[ \Lambda \cup \emptyset \cup \emptyset = \emptyset \]
\[ R_{14} = R_{14} \cup R_{12} R_{22}^* R_{24} \]
\[ \emptyset \cup \Lambda (1000^* \Lambda)^* \Lambda \]

\[ (1000^* \Lambda)^* \]
5. On the example of the automaton from Problem 4, explain, in detail, how the sequence 010101 will be presented as xyz according to the pumping lemma. For this sequence, check -- by tracing step-by-step -- that the sequence xy^2z for i = 2 is indeed accepted by the automaton.

So by pumping lemma there exists a p such that all of S have an L len(s) ≥ p in which there exists x, y, z such that S = xyz and len(y) > 0 and len(xy) ≤ p and ∀i (xy^iz ∈ L)

x is all before the first instance of a repetition
y is all after the first instance of repetition and before the second instance
z is all after the second instance of repetition

So now xy^2z

01010101

So xy^2z is accepted
6. Use the Pumping Lemma to prove that the language $L$ consisting of all repetitions $ww, www, \ldots$, is not regular.

$$L = \{ww, www, \ldots\}$$

$$L' = \{w^i \mid i \geq 2\}$$

Proof by contradiction.

Let's assume that $L$ is regular.

So by pumping lemma, there exists a $p$ such that $orall s \in L, \exists x, y, z$ such that $s = xyz$ and $\text{len}(y) > 0$ and $\text{len}(xy) \leq p$ and $\forall i (xy^iz \in L)$

Let's take $s = a^pb^pa^pb^p$ for $i = 2$ meaning $wh$.

If for $i = 2$ there's a contradiction then the whole language is not regular.

$$s = a^pb^pa^pb^p$$

the $\text{len}(s) = 4p$

$\text{len}(xy) \leq p$

meaning that $xy$ is in the first $2p$.

$xy \downarrow$

$xy^2z = \text{adding more as to the string by}$

pumping lemma shows that now the $\text{first word will be different than the}$

following words so we get a $\text{contradiction!}$

Therefore language $L$ is not regular.