1-2. Use a general algorithm to design a non-deterministic finite automaton recognizing the language \((1^* \cup 0)1\). After that, use the general algorithm to design a deterministic finite automaton recognizing this same language.
3. Use a general algorithm to transform the following finite automaton into the corresponding regular expression. This automaton has two states: a state q1 which is both a start state and a final state, and a state q2. In the state q1, 1 leads to q2, and 0 leads to q1. In the state q2, 1 leads to q2, and 0 leads to q1.

\[
\begin{align*}
D'_{S1} &= D_{S1} \cup (D_{S2} D_{22}^* D_{21}) = \Lambda \\
D'_{SF} &= D_{SF} \cup (D_{S2} D_{22}^* D_{21}) = \emptyset \\
D'_{11} &= D_{11} \cup (D_{12}^* D_{22} D_{21}) = \emptyset \cup (11^*0) \\
D'_{1F} &= D_{1F} \cup (D_{12} D_{22}^* D_{21}) = \Lambda \\
D'_{SF} &= D_{SF} \cup (D_{S1} D_{11}^* D_{1F}) = \left(\emptyset \cup (11^*0)\right)^* \\
\end{align*}
\]
4. On the example of the automaton from Problem 3, explain, in detail, how the sequence 010100 will be presented as xyz according to the pumping lemma. For this sequence, check -- by tracing step-by-step -- that the sequence xy^iz for i = 2 is indeed accepted by the automaton.
5. Use the Pumping Lemma to prove that the language $L$ consisting of all the words of the type $ww$ is not regular, where $a$ is a letter and $w$ can be any word. Here:

- if $w$ is an empty string, we get the word $a$,
- if $w$ is $ab$, we get $abaab$,

etc.

Proof by contradiction. Assume $L$ is regular. If $L$ is regular, there exists some number $p$ for all words accepted by $L$ such that the length of the word is $\geq p$, and there exists some concatenation of $xyz$ such that this concatenation represents the word, the length of $xy$ does not exceed $p$, $y$ is $\geq 0$, and $x y^i z$ is accepted by $L$ for all $i$, and $x y$ is found amongst the first $p$ symbols.

Consider words of the form $b^p a b^p$, a word that is accepted by $L$. The length of the word is $2p + 1$, which is $\geq p$. However, since $x \& y$ must be found amongst the first $p$ symbols, $y$ must necessarily consist only of the letter $b$. Thus, $x y^i z$ for $i > 1$ results in $b^q a b^p$, where $q$ is $> p$. This word is not accepted by $L$, which we assumed to be regular. However, by the Pumping Lemma, $b^q a b^p$ must be accepted by $L$. This is a contradiction, and thus, $L$ is not regular.
6. Use the general algorithm that we had in class to design a context-free grammar which generates exactly the words accepted by the automaton from Problem 3. Show how the word 010100 will be generated by this grammar.

\[
\begin{align*}
Q_1 &\rightarrow 0Q_2 \\
Q_1 &\rightarrow 1Q_2 \\
Q_2 &\rightarrow 1Q_2 \\
Q_2 &\rightarrow 0Q_1 \\
Q_1 &\rightarrow \epsilon
\end{align*}
\]

010100

\[
\begin{align*}
Q_1 &\rightarrow 0Q_2 \\
Q_1 &\rightarrow 01Q_2 \\
Q_2 &\rightarrow 01Q_1 \\
Q_2 &\rightarrow 0100Q_2 \\
Q_1 &\rightarrow 010100Q_1 \\
Q_2 &\rightarrow 010100
\end{align*}
\]
7. (For extra credit) Use the general algorithm to transform the grammar from Problem 6 into a pushdown automaton.