1-2. Use a general algorithm to design a non-deterministic finite automaton recognizing the language $(1 \cup 0)^*1$. After that, use the general algorithm to design a deterministic finite automaton recognizing this same language.
3. Use a general algorithm to transform the following finite automaton into the corresponding regular expression. This automaton has two states: a state q1 which is a start state, and a state q2 which is a final state. In the state q1, 0 leads to q2, and 1 leads to q1. In the state q2, 0 leads to q2, and 1 leads to q1.

\[
\begin{align*}
R_{S4} &= R_{S4} \cup R_{S4} R_{S2} \cdots R_{S2} \\
&= \Lambda \cup (\varnothing \cdots) = \Lambda \\
R'_{11} &= R_{11} \cup R_{11} R_{11} \cdots R_{11} \\
&= 1 \cup (0 \cdots) = 0 \cdots \\
\varnothing U (0 \cdots) &= 00^* \\
\emptyset U (\emptyset \cdots) &= \emptyset \\
\emptyset U (\emptyset \cdots) &= \emptyset \\
\end{align*}
\]

file:///Q:/cs3350.16a/test1.html
4. On the example of the automaton from Problem 3, explain, in detail, how the sequence 001100 will be presented as $xyz$ according to the pumping lemma. For this sequence, check -- by tracing step-by-step -- that the sequence $x^2z$ for $i = 2$ is indeed accepted by the automaton.

\[ \begin{align*}
X &= \text{before 1st repeating state} \\
Y &= \text{between 1st and 2nd rep.} \\
Z &= \text{after 2nd rep.}
\end{align*} \]

\[ x=0 \quad y=0 \quad z=1000 \]

The first repeating state is $q_2$.
So $x$ is what is before it which is $0$.$0$
$Y$ is what is between the two which is $0$
and $Z$ is what is after the rep which is $1000$

$x^2z = \text{is accepted}$
5. Use the Pumping Lemma to prove that the language $L$ consisting of all the words of the type $www$ is not regular, where $w$ can be any word. Here:

- if $w$ is an empty string, we get the word $\varepsilon$.
- if $w$ is $ab$, we get $ababab$.

etc.

$L = \varepsilon$ where $w$ is any word $\varepsilon$ is not regular

Proof by contradiction:

Let us assume that $L$ is regular then by pumping lemma there exist constants $m$ such that every word $s$ from $L$ whose length is $|s| \geq m$ can be represented as $s = xyz$, where $\text{len}(y) > 0$, $\text{len}(xy) \leq m$ and for every $i \geq 0$, $xy^iz \in L$.

Let take $w = a^m b^m$ then $s = a^{3m} b^{3m} a^m$. Here the length is $6m$, $m$ so by pumping lemma there exist constants $m$, $x$, $y$, $z$, since $s$ starts with $xy$ and $\text{len}(xy) \leq m$, this means that $m$ and $y$ are in the first set of $a's$, so when we take move we add $a$ to the first set of $a's$ but the # of $a's$ in the other sets does not change. So the # of $a's$ in the first set is no longer equal to $m$. $xy^iz \notin L$, a contradiction. By contradiction theorem our assumption is wrong so $L$ is not regular.
6. Use the general algorithm that we had in class to design a context-free grammar which generates exactly the words accepted by the automaton from Problem 3. Show how the word 001100 will be generated by this grammar.