1-2. Use a general algorithm to design a non-deterministic finite automaton recognizing the language \((1 \cup 0)^*1\). After that, use the general algorithm to design a deterministic finite automaton recognizing this same language.
3. Use a general algorithm to transform the following finite automaton into the corresponding regular expression. This automaton has two states: a state $q_1$ which is a start state, and a state $q_2$ which is a final state. In the state $q_1$, 0 leads to $q_2$, and 1 leads to $q_1$. In the state $q_2$, 0 leads to $q_2$, and 1 leads to $q_1$.

![Diagram of the automaton]

Eliminate $q_1$:

$$R'_{ij} = R_{ij} \cup (R_{ik} R_{kj} \ast R_{kj})$$

$$R'_{sF} = R_{sF} \cup (R_{sa} R_{a1} \ast R_{a1F})$$

$$R'_{q2F} = R_{q2F} \cup (R_{q21} R_{a1} \ast R_{a1F})$$

Eliminate $q_2$:

$$R'_{sF} = R_{sF} \cup (R_{sq2} R_{a2q} \ast R_{a2F})$$

$$= \emptyset \cup (1 \ast 0)$$

$$= 1 \ast 0$$

$$R'_{q2F} = R_{q2F} \cup (R_{q21} R_{a1} \ast R_{a1F})$$

$$= \emptyset \cup (1 \ast 0)$$

$$= (1 \ast 0)$$
4. On the example of the automaton from Problem 3, explain, in detail, how the sequence 001100 will be presented as xyz according to the pumping lemma. For this sequence, check -- by tracing step-by-step -- that the sequence xy^i z for i = 2 is indeed accepted by the automaton.

By pumping lemma, there exists a number \( p \) such that every word \( s \) from \( L \) whose length is \( \geq p \) can be represented as \( s = xyz \), where \( \text{len}(y) > 0 \), \( \text{len}(xy) \leq p \) and \( xy^i z \in L \) for every \( i = 0, 1, 2, \ldots \)

\[
L = 1^*0(0U(1)^*0)^* \\
S = 101011100 \\
= xyz
\]

\[
P = 2 \text{ states} \\
\text{len}(y) = 1 \\
\text{len}(xy) = 2 \leq p
\]

Sequence \( xy^2 z = xyyz \)

Sequence \( xyyz \) ends in final state 92
It is accepted by the automaton.
5. Use the Pumping Lemma to prove that the language $L$ consisting of all the words of the type $www$ is not regular, where $w$ can be any word. Here:

- if $w$ is an empty string, we get the word $A$
- if $w$ is $ab$, we get $ababab$

etc.

$L = www$ \{ $w$ is any word \} is not regular

Proof by contradiction. Let's assume $L$ is regular, then by pumping lemma, there exists a number $p$ such that every word $s$ from $L$ whose length is $2p$ can be represented as $s = xyz$, where $\text{len}(y) > 0$, $\text{len}(xy) \leq p$, and $xy^iz \in L$ for every $i = 0, 1, 2, ...$

Let's take $s = 0^p1^p0^p1^p0^p1^p$. Here, $\text{len}(s) = 6p \geq p$.

So, by pumping lemma, there exists corresponding $x, y, \text{and } z$, since $s$ starts with $xy$, and $\text{len}(y) \leq p$, this means that $x$ and $y$ are in $0^p$s, so when we take $xy^iz$, we add $0^p$s, but the number of $0^p$s in the other $2$ words does not change, so # of $0^p$s is no longer equal on every word:

$s = 0^p1^p0^p1^p0^p1^p \text{ does not belong to } L$

So, $xy^iz \notin L$. According to pumping lemma $xy^iz \notin L$ - a contradiction.

This contradiction shows that our assumption is wrong, so $L$ is not regular. The proposition is proven.
6. Use the general algorithm that we had in class to design a context-free grammar which generates exactly the words accepted by the automaton from Problem 3. Show how the word 001100 will be generated by this grammar.

\[ q_1 \rightarrow 1q_1 \]
\[ q_1 \rightarrow 0q_2 \]
\[ q_2 \rightarrow 1q_1 \]
\[ q_2 \rightarrow 0q_2 \]
\[ q_2 \rightarrow \lambda \]