1-2. Use a general algorithm to design a non-deterministic finite automaton recognizing the language $1(1^* \cup 0)$. After that, use the general algorithm to design a deterministic finite automaton recognizing this same language.
3-4. Use a general algorithm to transform the following finite automaton into the corresponding regular expression.

- This automaton has two states: a state $q_1$ which is a start state, and a state $q_2$ which is a final state.
- In the state $q_1$, 1 leads to $q_2$, and 0 leads to $q_1$.
- In the state $q_2$, 1 leads to $q_2$, and 0 leads to $q_1$.

1. Add new start and final

2. Delete $q_1$ \( \cup (00^*1) \)

\[
R_{s2}' = R_{s2} \cup (R_{s1} R_{s1}^* R_{12}) \\
= \emptyset \cup (\Lambda 0^* 1) \\
= 0^* 1 \\
R_{sf}' = R_{sf} \cup (R_{s1} R_{s1}^* R_{1f}) \\
= \emptyset \cup (\Lambda 0^* \emptyset) = \emptyset
\]

\[
R'_{22} = R_{22} \cup (R_{21} R_{12}^* R_{12}) \\
= 1 \cup (00^*1) \\
R'_{2f} = R_{2f} \cup (R_{21} R_{1f}^* R_{1f}) \\
= \Lambda \cup (00^*\emptyset) = \Lambda
\]
3. Delete q2

\[ R'_{SF} = R_{SF} \cup (R_{S2} R_{22^*} R_{21}) \]

\[ = \emptyset \cup (0^*1 (1U(00^*1))^*1) \]
3-4. Use a general algorithm to transform the following finite automaton into the corresponding regular expression.

- This automaton has two states: a state $q_1$ which is a start state, and a state $q_2$ which is a final state.
- In the state $q_1$, 1 leads to $q_2$, and 0 leads to $q_1$.
- In the state $q_2$, 1 leads to $q_2$, and 0 leads to $q_1$.

\[
R_{S} = R_{S1} \cup (R_{S2} \cdot R_{21}) \\
\land \cup (\emptyset \ldots ) = \emptyset
\]

\[
R'_{S1} = R_{S1} \cup (R_{S2} \cdot R_{21})
\]

\[
R'_{11} = R_{11} \cup (R_{12} \cdot R_{12})
\]

\[
R'_{12} = R_{12} \cup (R_{12} \cdot R_{12})
\]

\[
\emptyset \cup (1 1^* \emptyset) = 11^*
\]

\[
R'_{Sf} = R_{Sf} \cup (R_{S2} \cdot R_{2f})
\]

\[
\emptyset \cup (\emptyset \ldots ) \emptyset
\]

\[
R'_{Sf} = R_{Sf} \cup (R_{S2} \cdot R_{2f})
\]

\[
\emptyset \cup (\emptyset \cup (1 1^* 0)\ldots ) 11^*
\]

\[
(\emptyset \cup (1 1^* 0)\ldots ) (11^*)
\]
5. Let A be a language recognized by the automaton from Problem 3, and let B be the language corresponding to the following automaton:

- This automaton has 3 states, the starting state $q_1$, and the two final states $q_2$ and $q_3$.
- When you see 0, from $q_1$ you move to $q_2$, from $q_2$ you move to $q_3$, and from $q_3$ you move to $q_1$.
- When you see 1, from $q_1$ you move to $q_3$, from $q_2$ you move to $q_1$, and from $q_3$ you move to $q_2$.

Use the algorithm that we learned in class, with pairs of states from two automata as states of the new deterministic automaton, to describe the automata recognizing the union and the intersection of these two languages.