Homework Problem 17. Design a Turing machine that, given a positive binary number \( n \) greater than or equal to 2, subtracts 2 from this number. Test it, step-by-step, on the example of \( n = 2 \).

Solution. Here are the rules for the Turing machine:

- start, \( - \) \( \rightarrow \) R, skip
- skip, 0 \( \rightarrow \) R, moving
- skip, 1 \( \rightarrow \) R, moving
- moving, 0 \( \rightarrow \) 1, R
- moving, 1 \( \rightarrow \) 0, L, back
- back, 1 \( \rightarrow \) L
- back, 0 \( \rightarrow \) L
- back, \( - \) \( \rightarrow \) halt

Tracing. We start with the number \( 2_{10} = 10_2 \) which is represented as 01.

\[
\begin{array}{c|c|c|c}
\_ & 0 & 1 & \_ \_ \_ \\
\hline
\_ & 0 & 1 & \_ \_ \_ \\
\hline
\_ & 0 & 1 & \_ \_ \_ \\
\hline
\_ & 0 & 0 & \_ \_ \_ \\
\hline
\_ & 0 & 0 & \_ \_ \_ \\
\hline
\_ & 0 & 0 & \_ \_ \_ \\
\end{array}
\]

- start
- skip
- moving
- back
- back
- halt