Solution to Homework 7

**Task:** Show, step by step, how the grammar with rules $S \rightarrow \varepsilon$ and $S \rightarrow (S)$ will generate the word (()).

**Solution.** In this language, there is only one variable $S$, so this variable is a starting variable.

To this variable, we can apply either the first rule or the second rule. If we apply the rule $S \rightarrow \varepsilon$, then we get the empty string. So, to derive a non-empty word (()), we need to start with the second rule $S \rightarrow (S)$. So, we got the following derivation tree:

\[
\begin{array}{c}
S \\
( \\
S \\
) \\
( \\
S \\
( \\
S \\
()) \\
()) \\
()
\end{array}
\]

In the resulting word (S), we have a variable S. If we apply the rule $S \rightarrow \varepsilon$ to this word, we get the word (). Thus, to get a different word, we need to apply the second rule, i.e., get the derivation

$$S \rightarrow (S) \rightarrow ((S))$$

\[
\begin{array}{c}
S \\
( \\
S \\
()) \\
( \\
S \\
(() \\
S \\
()) \\
()) \\
()
\end{array}
\]

To the resulting word ((S)), we can apply each of the two rules. If we apply the first rule, we get exactly the desired word ((S)):

$$S \rightarrow (S) \rightarrow ((S)) \rightarrow ((S))$$