**Quiz 1 and Its Solution**

**Quiz.** Transform the following grammar with starting variable $S$ into Chomsky normal form:

$$S \to ASSA, \ S \to B, \ A \to a, \ B \to b.$$ 

**Solution.** First, on the preliminary stage, we add a new starting variable $S_0$ and a new rule $S_0 \to S$:

$$S \to ASSA, \ S \to B, \ A \to a, \ B \to b, \ S_0 \to S.$$ 

**Stage 0.** There are no non-Chomsky of the type $V \to \varepsilon$, so there is nothing to do on this stage.

**Stage 1.** We have two rules of the type $A \to B$: the rule $S \to B$ and the rule $S \to S_0$. Let us use the general algorithm to eliminate them one by one.

Let us first eliminate the rule $S \to B$. According to the algorithm, this means that we consider all the rules where $B$ goes to something, and add rules where $S$ goes to the same something. In our case, we only have one such rule $B \to b$, so we add the rule $S \to b$:

$$S \to ASSA, \ A \to a, \ B \to b, \ S_0 \to S, \ S \to b.$$ 

Now, we eliminate the rule $S_0 \to S$, we get:

$$S \to ASSA, \ A \to a, \ B \to b, \ S \to b, \ S_0 \to ASAA, \ S_0 \to b.$$ 

**Stage 2.** We introduce two new variables $V_a$ and $V_b$ and two new rules $V_a \to a$ and $V_b \to b$:

$$S \to ASSA, \ A \to a, \ B \to b, \ S \to b, \ S_0 \to ASAA, \ S_0 \to b, \ V_a \to a, \ V_b \to b.$$ 

**Comment.** In this particular cases, these two new rules are not needed, since no rules with right-hand side 2 or larger have terminal variables in the right-hand
side. However, no harm is done if we follow the general algorithm and add these two rules.

**Stage 3.** By using the general algorithm, we get the following rules:

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\begin{align*}
V_A S &\rightarrow A S, \quad V_A S S \rightarrow V_A S S, \quad S \rightarrow V_A S S A, \quad B \rightarrow b, \quad S \rightarrow b, \\
S_0 \rightarrow V_A S S A, \quad S_0 \rightarrow b, \quad V_a \rightarrow a, \quad V_b \rightarrow b.
\end{align*}
\]

This grammar is in Chomsky normal form.