Solution to Homework 7

**Task:** Show, step by step, how the grammar with rules $S \rightarrow \varepsilon$ and $S \rightarrow cSd$ will generate the word $ccdd$.

**Solution.** In this language, there is only one variable $S$, so this variable is a starting variable.

To this variable, we can apply either the first rule or the second rule. If we apply the rule $S \rightarrow \varepsilon$, then we get the empty string. So, to derive a non-empty word $ccdd$, we need to start with the second rule $S \rightarrow cSd$. So, we got the following derivation tree:

$$
\begin{array}{c}
S \\
\downarrow \hspace{1cm} \downarrow \\
c \hspace{1cm} S \\
\downarrow \\
d \\
\end{array}
$$

In the resulting word $cSd$, we have a variable $S$. If we apply the rule $S \rightarrow \varepsilon$ to this word, we get the word $cd$. Thus, to get a different word, we need to apply the second rule, i.e., get the derivation

$$S \rightarrow cSd \rightarrow ccSdd$$

$$
\begin{array}{c}
S \\
\downarrow \hspace{1cm} \downarrow \\
c \hspace{1cm} S \\
\downarrow \hspace{1cm} \downarrow \\
c \hspace{1cm} S \\
\downarrow \\
d \\
\end{array}
$$

To the resulting word $ccSdd$, we can apply each of the two rules. If we apply the first rule, we get exactly the desired word $ccdd$:

$$S \rightarrow cSd \rightarrow ccSdd \rightarrow ccdd$$