Solution to Problem 14

Task. For the $LL(1)$ grammar that we studied in class, with rules $S \rightarrow F$, $S \rightarrow (S + F)$, and $F \rightarrow a$, show how the word $((a + a) + a)$ can be represented as $uvwxyz$ in accordance with the pumping lemma for context-free grammars. Show that the corresponding word $uvxyz$ will be generated by this grammar.

Solution. The derivation of this string takes the following form:

In this derivation, we have two occurrences of the variable $F$, but they are not on the same branch. The lowest pair of occurrences of the same variable is the lowest pair of occurrences of the variable $S$: 
Thus, the desired decomposition of this word into $u$, $v$, $x$, $y$, and $z$ has the following form:

So, here $u = v = (., x = a, y = +a)$, and $z = +a)$. If we copy of the part between the two lowest occurrences of $S$ to the lower occurrence, we conclude that the word $uvvxyyz$ can be derived as follows: