Homework Problem 19. As we discuss in class, a Turing machine can be described as a finite automata with two stacks:

- the right stack contains, on top, the symbol to which the head points; below is the next symbol to the right, then the next to next symbol to the right, etc.;
- the left stack contains, on top, the symbol directly to the left of the head (if there is a one), under it is the next symbol to the left, etc.

On the example a Turing machine that computes \( n - 1 \) for a binary number \( n = 4 \), show, step-by-step:

- how each state of the corresponding Turing machine can be represented in terms of two stacks, and
- how each transition from one state to another can be implemented by push and pop operations.

Solution.

1. At first, we have the following configuration:

\[
\begin{array}{cccccccc}
- & 0 & 0 & 1 & - & - & \ldots & \text{start} \\
\end{array}
\]

Here, the left stack is empty, and the right stack has the form

\[
\begin{array}{cccccccc}
- & 0 & 0 & 1 \\
\end{array}
\]

2. Then, the configuration changes to:

\[
\begin{array}{cccccccc}
- & 0 & 0 & 1 & - & - & \ldots & \text{moving} \\
\end{array}
\]

Here, the two stacks have the following form:

\[
\begin{array}{cccccccc}
- & 0 \\
\end{array}
\]

To get to this configuration, we pop the symbol – (meaning black space) from the right stack and push it into the left stack.
3. Then, the configuration changes to:

```
- 1 0 1 - - ...  moving
```

Here, the left stack has the following form:

```
1
```

and the right stack has the following form:

```
0 1
```

To get to this configuration, we replace 0 with 1, pop 1 from the right stack and push it into the left stack.

4. Then, the configuration changes to:

```
- 1 1 1 - - ...  moving
```

Here, the left stack has the following form:

```
1 1
```

and the right stack has the following form:

```
-
```

```
1
```

To get to this configuration, we replace 0 with 1, pop 1 from the right stack and push it into the left stack.

5. Then, the configuration changes to:

```
- 1 1 1 0 - - ...  back
```

Here, the left stack has the following form:

```
1
```

and the right stack has the following form:

```
1 0
```

To get to this configuration, we replace 1 with 0, pop 1 from the left stack and push it into the right stack.

6. Then, the configuration changes to:

```
- 1 1 1 0 - - ...  back
```

Here, the left stack has the following form:

```
-
```

```
1 1
```

```
0
```

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To get to this configuration, we pop 1 from the left stack and push it into the right stack.

7. Then, the configuration changes to:

\[
\begin{array}{ccccccc}
- & 1 & 1 & 0 & - & - & \ldots \\
\end{array}
\]

back

Here, the left stack is empty, and the right stack has the following form:

\[
\begin{array}{ccc}
- \\
1 \\
1 \\
0 \\
\end{array}
\]

To get to this configuration, we pop – from the left stack and push it into the right stack.

8. Then, the configuration changes to:

\[
\begin{array}{ccccccc}
- & 1 & 1 & 0 & - & - & \ldots \\
\end{array}
\]

halt

Here, the contents of the tape did not change, and the location of the head did not change, so the stacks remain the same.