Solution to Homework 7

**Task:** Show, step by step, how the grammar with rules $S \rightarrow \varepsilon$ and $S \rightarrow +S -$ will generate the word $++--$.

**Solution.** In this language, there is only one variable $S$, so this variable is a starting variable.

To this variable, we can apply either the first rule or the second rule. If we apply the rule $S \rightarrow \varepsilon$, then we get the empty string. So, to derive a non-empty word $++--$, we need to start with the second rule $S \rightarrow +S -$. So, we got the following derivation tree:

```
  S
 / \  /
+   S-+
```

In the resulting word $+S-$, we have a variable $S$. If we apply the rule $S \rightarrow \varepsilon$ to this word, we get the word $+-$. Thus, to get a different word, we need to apply the second rule, i.e., get the derivation

$$S \rightarrow +S- \rightarrow + + S-$$

```
  S
 / \  /
+   S-+
   /  /
  +  S-
```

To the resulting word $++S-$, we can apply each of the two rules. If we apply the first rule, we get exactly the desired word $++--$:

$$S \rightarrow +S- \rightarrow + + S- \rightarrow + + --$$