Solution to Homework 8

**Background:** In Homework 3, we considered the following automaton. This automaton has two states: $s$ (= straight-A student) and $e$ (= everyone else); $s$ is the starting state, it is also the final state. The only two symbols are $A$ and $B$.

- From $s$, $A$ leads to $s$, and $B$ to $e$.
- From $e$, any symbol leads back to $e$.

This automaton has the following form:

![Automaton Diagram]

**Tasks:**

1. On the example of the automaton from Homework 3, show how the general algorithm will produce a context-free grammar that generates all the words accepted by this automaton – and only words generated by this automaton.

2. On the example of a word $AAA$ accepted by this automaton, show how the tracing of acceptance of this word by the finite automaton can be translated into a generation of this same word by your context-free grammar.

**Comment.** In CFG, terminal symbols are small letters, so we will use $a$ instead of $A$.

**Solution to Task 1.** The general algorithm for transforming FA into CFG is as follows:

- To each state $q$ of the FA, introduce a new variable $Q$.
- The variable corresponding to the starting state will be the starting variable of the new CFG.
- For each transition of the finite automaton
we add a rule $Q \rightarrow aQ'$.

- For each final state $f$ of the FA, we add a rule $F \rightarrow \varepsilon$.

By applying this general algorithm to this FA, we get a CFG with the starting variable $S$ and the following rules:

$$
S \rightarrow aS \\
S \rightarrow bE \\
E \rightarrow aE \\
E \rightarrow bE \\
S \rightarrow \varepsilon
$$

**Solution to Task 2.** Derivations in this grammar follow, step-by-step, the way the original finite automaton accepts a word. The word $AAA$ is accepted by the original finite automaton as follows:

- we start in the start state $s$; this corresponds to the starting variable $S$;
- then, we use the fact that once we are in the state $s$ and we see the symbol $A$, then we move to the state $s$; this transition corresponds to the rule $S \rightarrow aS$, so the generation so far is:
  $$S \rightarrow aS$$
- we have read all the symbols of the word, and we are in the final state $s$; for the FA, this means that the word $AAA$ is accepted; for CFG, we need to use the rule $S \rightarrow \varepsilon$ corresponding to the final state $s$; thus, we get the following derivation of the word $AAA$:
  $$S \rightarrow aS \rightarrow aaS \rightarrow aaaS \rightarrow aaaa.$$