1. Describe $\text{DIV}$ as a $\mu$-recursive function.

Let $p = m \text{DIV} n$.

It is known that $p \cdot n + r = m$, where $r$ stands for the remainder of the division.

For $r = 0$, $p \cdot n = m$; and for $r > 0$, $p \cdot n < m$. Therefore, $p \cdot n \leq m$.

To express $\text{DIV}$ using $\mu$-recursion, we need to find a relationship among $p$, $m$ and $n$ such that $p$ has the smallest value that will satisfy the $\text{DIV}$ properties.

Therefore, $\text{DIV}$ can be described by the following $\mu$-recursive function:

$$m \text{DIV} n = \mu p[(p + 1) \cdot n > m]$$

(1)

2. Show that the following function $f(n)$ is mu-recursive: $f(n) = 5$ for $n = 3$, $f(n) = 7$ for $n = 5$, and $f(n)$ is undefined for all other $n$.

Solution: $f(n) = \mu m.[(n = 3 \& m = 5) \lor (n = 5 \& m = 7)]$.

3. Describe a Turing Machine that computes the function $f(n) = n + 2$.

$$(\text{start, #}) \rightarrow \text{(go_right, R)}$$
$$(\text{go_right, 1}) \rightarrow \text{(go_right, R)}$$
$$(\text{go_right, #}) \rightarrow \text{(add_one, 1)}$$
$$(\text{add_one, 1}) \rightarrow \text{(go_right_again, R)}$$
$$(\text{go_right_again, #}) \rightarrow \text{(add_two, 1)}$$
$$(\text{go_right_again, 1}) \rightarrow \text{(go_right_again, R)}$$
$$(\text{add_two, 1}) \rightarrow \text{(back, L)}$$
$$(\text{back, 1}) \rightarrow \text{(back, L)}$$
$$(\text{back, #}) \rightarrow \text{halt}$$