7) Is there any set that is not decidable ?
{ <p, d> : p halts on d } → not decidable.
{ p : p always return 0 } → not decidable.

<u>Def</u> : A set 'A' is called recursively enumerable if there is an algorithm that eventually prints all elements of A.

```
n = 0;
while (true) {
        System.out.println(); {
}
```

24, 2009

if A is decidable + A decidable

$\bar{A} = \{ x ; x \notin A \}$

+ $\bar{A}$ is decidable

<u>Means</u> : there exists an algorithm that given n, checks whether $n \notin A$.

+ We want to prove : $\bar{A}$ is decidable.

→ we need an algorithm

$\xrightarrow{n} \boxed{} \longrightarrow$ we need an algorithm

in Compl. A (n)

$$\text{inCompl. A}(n) = \begin{cases} 1 & : \text{if } n \in \bar{A} \iff n \notin A. \\ 0 & ; \text{otherwise} \end{cases}$$

+ A decidable ⟺ ∃ algorithm, given a, can check whether $a \in A$.

+ A r.e ⟺ ∃ algorithm that eventually prints all elements of A.
(semi-decidable)

r.e = recursively enumerable !!

1) $\mathbb{N}$ is r.e.

2) $\emptyset$ is r.e.

3) Every decidable set is r.e

4) Finite sets are r.e since they are decidable.

5) Every cofinite set is r.e.

$\Updownarrow$

complement to a finite set !

6) If $\begin{cases} A \text{ is re.} \\ B \text{ is re} \end{cases} \Rightarrow A \cup B$ is r.e.

Since A & B are not finite then we have to do

- Print in A for 1 hour
- ______ B ______
- ______ A ______ more

...

...

we can print all elements in A, B !!

$\Rightarrow A \cup B$ is r.e.

7) If A is r.e. and B is re

then $A \cap B$ is r.e ?

- run in A for 1 hr (make a partial list)

run in B for 1 hr (______)

print all common elements

- run in A for 1 hr more (repeating)

...

...

8) If A is R.e and $\overline{A}$ is R.e.

$\rightarrow$ then A is decidable.

Algorithm,

run in A for 1-hr
— in $\bar{A}$ for 1-hr
— ...
— ...

we stop when n appears in one of the lists.

9) If A is r.e, then $\bar{A}$ is r.e ?? NO WAY

Recall: we had an example of a set which is not decidable.

$H = \{ (p, d) ; p \text{ halts and } \}$

H is r.e but not decidable !! — can be printed / can not check if (p,d) in H

(S1) Run program #0 and ~~0~~#1 for 1 hr. on data 0, 1.

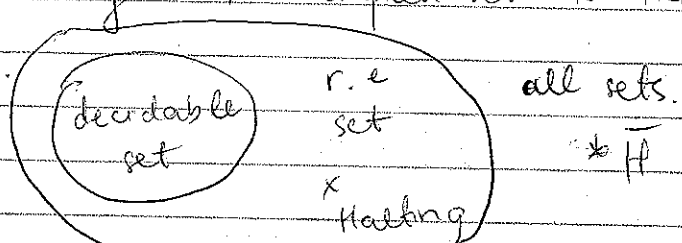
(S2) if halts print (p, d).
run programs 0, 1 and 2.
for 2 hrs. on data 0, 1, 2.

...

(S3) ...

10) There exists a set A which is r.e but not decidable.

11) Is ~~there~~ every ~~any~~ set r.e ?
No, there is a set which is not r.e
$\overline{\text{Halting}}$ // complement to Halting set.



$$\text{in}A(n) = \begin{cases} 1 : & \text{if } n \in A \\ 0 , & \text{otherwise} \end{cases}$$ || decidable.

<u>Semidecidable</u> :

$$\text{in } A(n) = \begin{cases} 1 : & \text{if } n \in A \\ \emptyset & \text{if } n \notin A \end{cases}$$

(runs indefinitely)

+ If A is r.e → then A is semi-decidable
  - Have an algorithm that prints all elements in A.

inA (n) waits and checks every hour whether n was printed.

halts.

true (1) → $n \in A$

runs definitely → In A(n)

+ If A is semi-decidable then A is r.e.