7) Is there any set that is not decidable ?

{ <p, d> : p halts on d } → not decidable.

{ p : p always return 0 } → not decidable.

Def: A set A is called recursively enumerable if there is an algorithm that eventually prints all elements of A

```
n = 0;
while (true) {
    System.out.println();
}
```

24, 2019

if A is decidable ⇒ $\bar{A}$ decidable

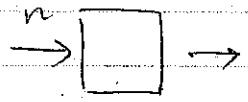
$\bar{A} = \{ x ; x \notin A \}$

⇐ $\bar{A}$ is decidable

Means: there exists an algorithm that given n, checks whether $n \notin A$.

⇐ We want to prove: $\bar{A}$ is decidable.

→ we need an algorithm



we need an algorithm

inCompl_A(n)

$$\text{inCompl}_A(n) = \begin{cases} 1 & ; \text{ if } n \in \bar{A} \Leftrightarrow n \notin A \\ 0 & ; \text{ otherwise} \end{cases}$$

⇐ A decidable ⇔ ∃ algorithm, given a, can check whether $a \in A$.

⇐ A r.e ⇔ ∃ algorithm that eventually prints (semi-decidable) all elements of A.

r.e = recursively enumerable !!

1) $\mathbb{N}$ is r.e.
2) ϕ is r.e.
3) Every decidable set is r.e
4) Finite sets are r.e since they are decidable.
5) Every cofinite set is r.e.
   ↕ complement to a finite set !!
6) If $\begin{cases} A \text{ is re.} \\ B \text{ is re} \end{cases} \Rightarrow A \cup B$ is r.e.

   Since A & B are not finite then we have to do
   - Print in A for 1 hour
   - ——— B ———
   - ——— A ——— more
   ...
   ...

   we can print all elements in A, B !!
   ⇒ $A \cup B$ is r.e.
7) If A is r.e. and B is re
   then $A \cap B$ is r.e ?

- run in A for 1 hr (make a partial list)
  run in B for 1 hr (———)
  print all common elements

- run in A for 1 hr more (repeating)
  ...
  ...
8) If A is Re and $\overline{A}$ is R.e.
   → then A is decidable

Algorithm:

run in A for 1 hr	we stop when n appears in one of the lists.
— in $\bar{A}$ for 1 hr	
— ...	
— ...	

9) If A is r.e, then $\bar{A}$ is r.e ?? NO WAY

Recall: we had an example of a set which is not decidable.

$H = \{(p, d) \nmid$; p halts and $\nmid$

H is r.e but not decidable !! ← can be printed; can not check if $(p,d) \in H$ / $\notin H$

(S1) Run program #0 and #1 for 1 hr. on data 0, 1.

(S2) if halts print (p, d).
run programs 0, 1 and 2.
for 2 hrs. on data 0, 1, 2.

...

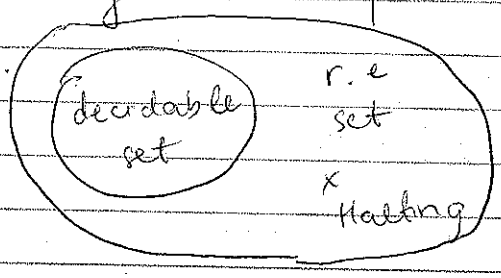
(S3) ...

10) There exists a set A which is r.e but not decidable.

11) Is ~~there~~ every set r.e ?

No, there is a set which is not r.e.

Halting // complement to Halting set.



$$inA(n) := \begin{cases} 1 & : \text{if } n \in A \\ 0 & : \text{otherwise} \end{cases}$$ || decidable.

Semi-decidable :

$$inA(n) = \begin{cases} 1 & : \text{if } n \in A \\ \emptyset & \text{if } n \notin A \end{cases}$$

↓ (runs indefinitely)

+ If A is r.e → then A is semi-decidable
- Have an algorithm that prints all elements in A.

inA(n) waits and checks every hour whether n was printed.

halts.
↗ true (1) → $n \in A$
↘ runs definitely → $inA(n)$

+ If A is semi-decidable then A is r.e.