7) Is there any set that is not decidable?

- \( \langle p, q \rangle \) is decidable only if \( p \) always returns \( 0 \) \( \rightarrow \) not decidable
- \( p \) is decidable \( \rightarrow \) not decidable

**Def.** A set \( A \) is called recursively enumerable if there is an algorithm that eventually prints all elements of \( A \).

\[
\begin{align*}
& n = 0 \\
& \text{while (true) } \\
& \quad \text{System.out.println( );} \\
& \quad y
\end{align*}
\]

if \( A \) is decidable + \( A \) decidable

\[ A = \{ x ; x \in A \} \]

\[ + A \text{ is decidable} \]

Means: there exists an algorithm that given \( n \), checks whether \( n \in A \).

\[ + \text{We want to prove}: \bar{A} \text{ is decidable} \]

\[ + \text{we need an algorithm} \]

\[ n \rightarrow \Box \rightarrow \text{we need an algorithm} \]

\[ \text{incomp.} A(n) \]

\[ \text{incomp.} A(n) = \begin{cases} 1 & \text{if } n \in \bar{A} \Rightarrow n \notin A \\ 0 & \text{otherwise} \end{cases} \]

\[ + A \text{ decidable } \Rightarrow \exists \text{ algorithm}, \text{given } a, \text{ can check whether } a \in \bar{A} \]

\[ + A \text{ (e) } \Rightarrow \exists \text{ algorithm that eventually prints } (\text{semi-decidable}) \text{ all elements of } A \]
\( r.e. = \text{recursively enumerable} \)

1. \( \mathbb{N} \) is r.e.
2. \( \emptyset \) is r.e.
3. Every decidable set is r.e.
4. Finite sets are r.e. since they are decidable.
5. Every infinite set is r.e.
6. If \( A \) is r.e. \( \Rightarrow \) \( A \cup B \) is r.e.
7. If \( A \) is r.e. and \( B \) is r.e.
   then \( A \cap B \) is r.e.
   - run in \( A \) for 1 hr (make a partial list)
   - run in \( B \) for 1 hr (partial)
   - print all common elements
   - run in \( A \) for 1 hr more (repeating)

8. If \( A \) is r.e. and \( \overline{A} \) is r.e.
   then \( A \) is decidable.
Algorithm:

run in $A$ for 1 hr

$\neg$ run in $A$ for 1 hr

we stop when $n$

appears in one of

the lists.

9) If $A$ is r.e., then $\neg A$ is r.e. 22 NO WAY

Recall: we had an example of a set which is not decidable:

$H = \{(p, d) | p$ halts and \}

$H$ is r.e. but not decidable. !< can not check if $(1, 0) \in H$

S1) Run programs 0 and 1

for 1 hr on data 0, 1.

S2) If halts print $(p, d)$

run programs 2, 3, and 4.

for 2 hrs on data 0, 1, 2.

S3

10) There exists a set $A$ which is r.e. but not decidable

11) Is there any set r.e.?

No, there is a set which is not r.e.

$\neg$ Halting \// complement to Halting set.

\[ \neg \text{decidable set} \quad \text{r.e. set} \quad \text{all sets.} \]

\[ \text{Halting} \quad \text{all sets.} \]
\[ m_A(n) = \begin{cases} 1 & \text{if } n \in A \\ 0 & \text{otherwise} \end{cases} \]

Semidecider:

\[ m_A(n) = \begin{cases} \emptyset & \text{if } n \notin A \\ \text{(runs indefinitely)} & \text{if } n \in A \end{cases} \]

If \( A \) is r.e. \( \implies \) \( m_A(n) \) is a semi-decidable
- Have an algorithm that prints all elements in \( A \).

- \( m_A(n) \) waits and checks every hour \( \delta \) whether \( n \) was printed.

- If \( m_A(n) \) halts: \( \text{true} \) \( \implies \) \( n \in A \)

- \( \text{runs indefinitely} \implies m_A(n) \)

If \( A \) is semi-decidable then \( A \) is r.e.